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TABLES

FOR FACILITATING THE USE OF

HARMONIC ANALYSIS

92

AS ARRANGED BY

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TABLES FOR FACILITATING HARMONIC ANALYSIS

If we arrange a set of numbers

$$a_1, a_2, a_3 \ldots a_n, a_{n+1} \ldots$$

representing the values of a quantity at equal intervals of time (e.g. the total rainfalls in consecutive months) in series of n terms (either singly or as means of several series), then to find the expression

$$s \sin \theta + c \cos \theta$$

representing them most closely, where $n\theta = 2\pi$, we form

$$S = o + a_2 \sin \theta + a_3 \sin 2\theta + \ldots + a_n \sin (n-1)\theta$$
$$C = a_1 + a_2 \cos \theta + a_3 \cos 2\theta + \ldots + a_n \cos (n-1)\theta$$

and then

$$s' = 2\,S/n, \quad c = 2\,C/n$$

The following tables are arranged to allow the products $a_{r+1} \sin r\theta$ and $a_{r+1} \cos r\theta$ to be taken out at sight, to two figures, which suffice for much of the exploring work represented by Professor Schuster's periodogram method.

Explanation is scarcely necessary. On the left is given the value of a_{r+1}: and on the same line, in the column headed $r+1$, is given the product $a_{r+1} \sin r\theta$ or $a_{r+1} \cos r\theta$, as the case may be. If $\sin r\theta$ or $\cos r\theta$ is negative this is indicated by attaching the minus sign to $r+1$ in the heading (or at the foot).

Note that, to keep corresponding tables for 11 terms at the same opening, the short tables for 10 and 12 terms are put together.

An example of the use of the tables is given at the end.

9 TERMS — SINE θ

1 *is zero*

	+2	+3	+4	+5
Nos.	·643	·985	·866	·342
1	1	1	1	0
2	1	2	2	1
3	2	3	3	1
4	3	4	3	1
5	3	5	4	2
6	4	6	5	2
7	5	7	6	2
8	5	8	7	3
9	6	9	8	3
10	6	10	9	3
11	7	11	10	4
12	8	12	10	4
13	8	13	11	4
14	9	14	12	5
15	10	15	13	5
16	10	16	14	5
17	11	17	15	6
18	12	18	16	6
19	12	19	16	6
20	13	20	17	7
21	14	21	18	7
22	14	22	19	8
23	15	23	20	8
24	15	24	21	8
25	16	25	22	9
26	17	26	23	9
27	17	27	23	9
28	18	28	24	10
29	19	29	25	10
30	19	30	26	10
31	20	31	27	11
32	21	32	28	11
33	21	33	29	11
34	22	33	29	12
35	23	34	30	12
36	23	35	31	12
37	24	36	32	13
38	24	37	33	13
39	25	38	34	13
40	26	39	35	14
41	26	40	36	14
42	27	41	36	14
43	28	42	37	15
44	28	43	38	15
45	29	44	39	15
46	30	45	40	16
47	30	46	41	16
48	31	47	42	16
49	32	48	42	17
50	32	49	43	17
51	33	50	44	17
52	33	51	45	18
53	34	52	46	18
54	35	53	47	18
55	35	54	48	19
56	36	55	48	19
57	37	56	49	19
58	37	57	50	20
59	38	58	51	20
60	39	59	52	21
61	39	60	53	21
62	40	61	54	21
63	41	62	55	22
64	41	63	55	22
65	42	64	56	22
66	42	65	57	23
67	43	66	58	23
68	44	67	59	23
69	44	68	60	24
70	45	69	61	24
71	46	70	61	24
72	46	71	62	25
73	47	72	63	25
74	48	73	64	25
75	48	74	65	26
76	49	75	66	26
77	50	76	67	26
78	50	77	68	27
79	51	78	68	27
80	51	79	69	27
81	52	80	70	28
82	53	81	71	28
83	53	82	72	28
84	54	83	73	29
85	55	84	74	29
86	55	85	74	29
87	56	86	75	30
88	57	87	76	30
89	57	88	77	30
90	58	89	78	31
91	59	90	79	31
92	59	91	80	31
93	60	92	81	32
94	60	93	81	32
95	61	94	82	32
96	62	95	83	33
97	62	96	84	33
98	63	97	85	34
99	64	98	86	34
100	64	99	87	34
	−9	−8	−7	−6

9 TERMS — COS θ

1 *is* 1·000

	+2	+3	−4	−5
Nos.	·766	·174	·500	·940
1	1	0	1	1
2	2	0	1	2
3	2	1	2	3
4	3	1	2	4
5	4	1	3	5
6	5	1	3	6
7	5	1	4	7
8	6	1	4	8
9	7	2	5	8
10	8	2	5	9
11	8	2	6	10
12	9	2	6	11
13	10	2	7	12
14	11	2	7	13
15	11	3	8	14
16	12	3	8	15
17	13	3	9	16
18	14	3	9	17
19	15	3	10	18
20	15	3	10	19
21	16	4	11	20
22	17	4	11	21
23	18	4	12	22
24	18	4	12	23
25	19	4	13	24
26	20	5	13	24
27	21	5	14	25
28	21	5	14	26
29	22	5	15	27
30	23	5	15	28
31	24	5	16	29
32	25	6	16	30
33	25	6	17	31
34	26	6	17	32
35	27	6	18	33
36	28	6	18	34
37	28	6	19	35
38	29	7	19	36
39	30	7	20	37
40	31	7	20	38
41	31	7	21	39
42	32	7	21	39
43	33	7	22	40
44	34	8	22	41
45	34	8	23	42
46	35	8	23	43
47	36	8	24	44
48	37	8	24	45
49	38	9	25	46
50	38	9	25	47
	+9	+8	−7	−6

	+2	+3	−4	−5
Nos.	·766	·174	·500	·940
51	39	9	26	48
52	40	9	26	49
53	41	9	27	50
54	41	9	27	51
55	42	10	28	52
56	43	10	28	53
57	44	10	29	54
58	44	10	29	55
59	45	10	30	55
60	46	10	30	56
61	47	11	31	57
62	47	11	31	58
63	48	11	32	59
64	49	11	32	60
65	50	11	33	61
66	51	11	33	62
67	51	12	34	63
68	52	12	34	64
69	53	12	35	65
70	54	12	35	66
71	54	12	36	67
72	55	13	36	68
73	56	13	37	69
74	57	13	37	70
75	57	13	38	71
76	58	13	38	71
77	59	13	39	72
78	60	14	39	73
79	61	14	40	74
80	61	14	40	75
81	62	14	41	76
82	63	14	41	77
83	64	14	42	78
84	64	15	42	79
85	65	15	43	80
86	66	15	43	81
87	67	15	44	82
88	67	15	44	83
89	68	15	45	84
90	69	16	45	85
91	70	16	46	86
92	70	16	46	86
93	71	16	47	87
94	72	16	47	88
95	73	17	48	89
96	74	17	48	90
97	74	17	49	91
98	75	17	49	92
99	76	17	50	93
100	77	17	50	94
	+9	+8	−7	−6

10 TERMS — SINES AND COSINES

1 is zero
6 is zero

1 is 1·000 +
6 is 1·000 −

	Sine.			Sine.			Cos.			Cos.	
+	2 & 5	3 & 4	+	2 & 5	3 & 4	+	2 & 10	3 & 9	+	2 & 10	3 & 9
Nos.	·588	·951	Nos.	·588	·951	Nos.	·809	·309	Nos.	·809	·309
1	1	1	**51**	30	49	**1**	1	0	**51**	41	16
2	1	2	**52**	31	49	**2**	2	1	**52**	42	16
3	2	3	**53**	31	50	**3**	2	1	**53**	43	16
4	2	4	**54**	32	51	**4**	3	1	**54**	44	17
5	3	5	**55**	32	52	**5**	4	2	**55**	44	17
6	4	6	**56**	33	53	**6**	5	2	**56**	45	17
7	4	7	**57**	34	54	**7**	6	2	**57**	46	18
8	5	8	**58**	34	55	**8**	6	2	**58**	47	18
9	5	9	**59**	35	56	**9**	7	3	**59**	48	18
10	6	10	**60**	35	57	**10**	8	3	**60**	49	19
11	6	11	**61**	36	58	**11**	9	3	**61**	49	19
12	7	11	**62**	36	59	**12**	10	4	**62**	50	19
13	8	12	**63**	37	60	**13**	11	4	**63**	51	19
14	8	13	**64**	38	61	**14**	11	4	**64**	52	20
15	9	14	**65**	38	62	**15**	12	5	**65**	53	20
16	9	15	**66**	39	63	**16**	13	5	**66**	53	20
17	10	16	**67**	39	64	**17**	14	5	**67**	54	21
18	11	17	**68**	40	65	**18**	15	6	**68**	55	21
19	11	18	**69**	41	66	**19**	15	6	**69**	56	21
20	12	19	**70**	41	67	**20**	16	6	**70**	57	22
21	12	20	**71**	42	68	**21**	17	6	**71**	57	22
22	13	21	**72**	42	68	**22**	18	7	**72**	58	22
23	14	22	**73**	43	69	**23**	19	7	**73**	59	23
24	14	23	**74**	44	70	**24**	19	7	**74**	60	23
25	15	24	**75**	44	71	**25**	20	8	**75**	61	23
26	15	25	**76**	45	72	**26**	21	8	**76**	61	23
27	16	26	**77**	45	73	**27**	22	8	**77**	62	24
28	16	27	**78**	46	74	**28**	23	9	**78**	63	24
29	17	28	**79**	46	75	**29**	23	9	**79**	64	24
30	18	29	**80**	47	76	**30**	24	9	**80**	65	25
31	18	29	**81**	48	77	**31**	25	10	**81**	66	25
32	19	30	**82**	48	78	**32**	26	10	**82**	66	25
33	19	31	**83**	49	79	**33**	27	10	**83**	67	26
34	20	32	**84**	49	80	**34**	28	11	**84**	68	26
35	21	33	**85**	50	81	**35**	28	11	**85**	69	26
36	21	34	**86**	51	82	**36**	29	11	**86**	70	27
37	22	35	**87**	51	83	**37**	30	11	**87**	70	27
38	22	36	**88**	52	84	**38**	31	12	**88**	71	27
39	23	37	**89**	52	85	**39**	32	12	**89**	72	28
40	24	38	**90**	53	86	**40**	32	12	**90**	73	28
41	24	39	**91**	54	87	**41**	33	13	**91**	74	28
42	25	40	**92**	54	87	**42**	34	13	**92**	74	28
43	25	41	**93**	55	88	**43**	35	13	**93**	75	29
44	26	42	**94**	55	89	**44**	36	14	**94**	76	29
45	26	43	**95**	56	90	**45**	36	14	**95**	77	29
46	27	44	**96**	56	91	**46**	37	14	**96**	78	30
47	28	45	**97**	57	92	**47**	38	15	**97**	78	30
48	28	46	**98**	58	93	**48**	39	15	**98**	79	30
49	29	47	**99**	58	94	**49**	40	15	**99**	80	31
50	29	48	**100**	59	95	**50**	40	15	**100**	81	31
—	7 & 10	8 & 9	—	7 & 10	8 & 9	—	5 & 7	4 & 8	—	5 & 7	4 & 8

12 TERMS — SINES AND COSINES

1 *and* **7** *are zero*
4 *and* **10** *are* 1·000

1 *and* **7** *are* 1·000
4 *and* **10** *are zero*

	Sine.			Sine.			Cos.			Cos.	
+	**2 & 6**	**3 & 5**	**+**	**2 & 6**	**3 & 5**	**+**	**2 & 12**	**3 & 11**	**+**	**2 & 12**	**3 & 11**
Nos.	·500	·866	Nos.	·500	·866	Nos.	·866	·500	Nos.	·866	·500
1	1	1	**51**	26	44	**1**	1	1	**51**	44	26
2	1	2	**52**	26	45	**2**	2	1	**52**	45	26
3	2	3	**53**	27	46	**3**	3	2	**53**	46	27
4	2	3	**54**	27	47	**4**	3	2	**54**	47	27
5	3	4	**55**	28	48	**5**	4	3	**55**	48	28
6	3	5	**56**	28	48	**6**	5	3	**56**	48	28
7	4	6	**57**	29	49	**7**	6	4	**57**	49	29
8	4	7	**58**	29	50	**8**	7	4	**58**	50	29
9	5	8	**59**	30	51	**9**	8	5	**59**	51	30
10	5	9	**60**	30	52	**10**	9	5	**60**	52	30
11	6	10	**61**	31	53	**11**	10	6	**61**	53	31
12	6	10	**62**	31	54	**12**	10	6	**62**	54	31
13	7	11	**63**	32	55	**13**	11	7	**63**	55	32
14	7	12	**64**	32	55	**14**	12	7	**64**	55	32
15	8	13	**65**	33	56	**15**	13	8	**65**	56	33
16	8	14	**66**	33	57	**16**	14	8	**66**	57	33
17	9	15	**67**	34	58	**17**	15	9	**67**	58	34
18	9	16	**68**	34	59	**18**	16	9	**68**	59	34
19	10	16	**69**	35	60	**19**	16	10	**69**	60	35
20	10	17	**70**	35	61	**20**	17	10	**70**	61	35
21	11	18	**71**	36	61	**21**	18	11	**71**	61	36
22	11	19	**72**	36	62	**22**	19	11	**72**	62	36
23	12	20	**73**	37	63	**23**	20	12	**73**	63	37
24	12	21	**74**	37	64	**24**	21	12	**74**	64	37
25	13	22	**75**	38	65	**25**	22	13	**75**	65	38
26	13	23	**76**	38	66	**26**	23	13	**76**	66	38
27	14	23	**77**	39	67	**27**	23	14	**77**	67	39
28	14	24	**78**	39	68	**28**	24	14	**78**	68	39
29	15	25	**79**	40	68	**29**	25	15	**79**	68	40
30	15	26	**80**	40	69	**30**	26	15	**80**	69	40
31	16	27	**81**	41	70	**31**	27	16	**81**	70	41
32	16	28	**82**	41	71	**32**	28	16	**82**	71	41
33	17	29	**83**	42	72	**33**	29	17	**83**	72	42
34	17	29	**84**	42	73	**34**	29	17	**84**	73	42
35	18	30	**85**	43	74	**35**	30	18	**85**	74	43
36	18	31	**86**	43	74	**36**	31	18	**86**	74	43
37	19	32	**87**	44	75	**37**	32	19	**87**	75	44
38	19	33	**88**	44	76	**38**	33	19	**88**	76	44
39	20	34	**89**	45	77	**39**	34	20	**89**	77	45
40	20	35	**90**	45	78	**40**	35	20	**90**	78	45
41	21	36	**91**	46	79	**41**	36	21	**91**	79	46
42	21	36	**92**	46	80	**42**	36	21	**92**	80	46
43	22	37	**93**	47	81	**43**	37	22	**93**	81	47
44	22	38	**94**	47	81	**44**	38	22	**94**	81	47
45	23	39	**95**	48	82	**45**	39	23	**95**	82	48
46	23	40	**96**	48	83	**46**	40	23	**96**	83	48
47	24	41	**97**	49	84	**47**	41	24	**97**	84	49
48	24	42	**98**	49	85	**48**	42	24	**98**	85	49
49	25	42	**99**	50	86	**49**	42	25	**99**	86	50
50	25	43	**100**	50	87	**50**	43	25	**100**	87	50
—	**8 & 12**	**9 & 11**	**—**	**8 & 12**	**9 & 11**	**—**	**6 & 8**	**5 & 9**	**—**	**6 & 8**	**5 & 9**

11 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6
Nos.	·541	·910	·990	·756	·282
1	1	1	1	1	0
2	1	2	2	2	1
3	2	3	3	2	1
4	2	4	4	3	1
5	3	5	5	4	1
6	3	5	6	5	2
7	4	6	7	5	2
8	4	7	8	6	2
9	5	8	9	7	3
10	5	9	10	8	3
11	6	10	11	8	3
12	6	11	12	9	3
13	7	12	13	10	4
14	8	13	14	11	4
15	8	14	15	11	4
16	9	15	16	12	5
17	9	15	17	13	5
18	10	16	18	14	5
19	10	17	19	14	5
20	11	18	20	15	6
21	11	19	21	16	6
22	12	20	22	17	6
23	12	21	23	17	6
24	13	22	24	18	7
25	14	23	25	19	7
26	14	24	26	20	7
27	15	25	27	20	8
28	15	25	28	21	8
29	16	26	29	22	8
30	16	27	30	23	8
31	17	28	31	23	9
32	17	29	32	24	9
33	18	30	33	25	9
34	18	31	34	26	10
35	19	32	35	26	10
36	19	33	36	27	10
37	20	34	37	28	10
38	21	35	38	29	11
39	21	35	39	29	11
40	22	36	40	30	11
41	22	37	41	31	12
42	23	38	42	32	12
43	23	39	43	33	12
44	24	40	44	33	12
45	24	41	45	34	13
46	25	42	46	35	13
47	25	43	47	36	13
48	26	44	48	36	14
49	27	45	49	37	14
50	27	46	50	38	14
51	28	46	50	39	14
52	28	47	51	39	15
53	29	48	52	40	15
54	29	49	53	41	15
55	30	50	54	42	16
56	30	51	55	42	16
57	31	52	56	43	16
58	31	53	57	44	16
59	32	54	58	45	17
60	32	55	59	45	17
61	33	56	60	46	17
62	34	56	61	47	17
63	34	57	62	48	18
64	35	58	63	48	18
65	35	59	64	49	18
66	36	60	65	50	19
67	36	61	66	51	19
68	37	62	67	51	19
69	37	63	68	52	19
70	38	64	69	53	20
71	38	65	70	54	20
72	39	66	71	54	20
73	39	66	72	55	21
74	40	67	73	56	21
75	41	68	74	57	21
76	41	69	75	57	21
77	42	70	76	58	22
78	42	71	77	59	22
79	43	72	78	60	22
80	43	73	79	60	23
81	44	74	80	61	23
82	44	75	81	62	23
83	45	76	82	63	23
84	45	76	83	64	24
85	46	77	84	64	24
86	47	78	85	65	24
87	47	79	86	66	25
88	48	80	87	67	25
89	48	81	88	67	25
90	49	82	89	68	25
91	49	83	90	69	26
92	50	84	91	70	26
93	50	85	92	70	26
94	51	86	93	71	27
95	51	86	94	72	27
96	52	87	95	73	27
97	52	88	96	73	27
98	53	89	97	74	28
99	54	90	98	75	28
100	54	91	99	76	28
—	11	10	9	8	7

11 TERMS — COS θ

1 *is* 1·000

	+2	+3	−4	−5	−6
Nos.	·841	·416	·142	·655	·960
1	1	0	0	1	1
2	2	1	0	1	2
3	3	1	0	2	3
4	3	2	1	3	4
5	4	2	1	3	5
6	5	2	1	4	6
7	6	3	1	5	7
8	7	3	1	5	8
9	8	4	1	6	9
10	8	4	1	7	10
11	9	5	2	7	11
12	10	5	2	8	12
13	11	5	2	9	12
14	12	6	2	9	13
15	13	6	2	10	14
16	13	7	2	10	15
17	14	7	2	11	16
18	15	7	3	12	17
19	16	8	3	12	18
20	17	8	3	13	19
21	18	9	3	14	20
22	19	9	3	14	21
23	19	10	3	15	22
24	20	10	3	16	23
25	21	10	4	16	24
26	22	11	4	17	25
27	23	11	4	18	26
28	24	12	4	18	27
29	24	12	4	19	28
30	25	12	4	20	29
31	26	13	4	20	30
32	27	13	5	21	31
33	28	14	5	22	32
34	29	14	5	22	33
35	29	15	5	23	34
36	30	15	5	24	35
37	31	15	5	24	36
38	32	16	5	25	36
39	33	16	6	26	37
40	34	17	6	26	38
41	34	17	6	27	39
42	35	17	6	28	40
43	36	18	6	28	41
44	37	18	6	29	42
45	38	19	6	29	43
46	39	19	7	30	44
47	40	20	7	31	45
48	40	20	7	31	46
49	41	20	7	32	47
50	42	21	7	33	48
51	43	21	7	33	49
52	44	22	7	34	50
53	45	22	8	35	51
54	45	22	8	35	52
55	46	23	8	36	53
56	47	23	8	37	54
57	48	24	8	37	55
58	49	24	8	38	56
59	50	25	8	39	57
60	50	25	9	39	58
61	51	25	9	40	59
62	52	26	9	41	60
63	53	26	9	41	60
64	54	27	9	42	61
65	55	27	9	43	62
66	56	27	9	43	63
67	56	28	10	44	64
68	57	28	10	45	65
69	58	29	10	45	66
70	59	29	10	46	67
71	60	30	10	47	68
72	61	30	10	47	69
73	61	30	10	48	70
74	62	31	11	48	71
75	63	31	11	49	72
76	64	32	11	50	73
77	65	32	11	50	74
78	66	32	11	51	75
79	66	33	11	52	76
80	67	33	11	52	77
81	68	34	12	53	78
82	69	34	12	54	79
83	70	35	12	54	80
84	71	35	12	55	81
85	71	35	12	56	82
86	72	36	12	56	83
87	73	36	12	57	84
88	74	37	12	58	84
89	75	37	13	58	85
90	76	37	13	59	86
91	77	38	13	60	87
92	77	38	13	60	88
93	78	39	13	61	89
94	79	39	13	62	90
95	80	40	13	62	91
96	81	40	14	63	92
97	82	40	14	64	93
98	82	41	14	64	94
99	83	41	14	65	95
100	84	42	14	66	96
	+11	+10	−9	−8	−7

13 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6	7	+	2	3	4	5	6	7
Nos.	·465	·823	·993	·935	·663	·239	Nos.	·465	·823	·993	·935	·663	·239
1	0	1	1	1	1	0	**51**	24	42	51	48	34	12
2	1	2	2	2	1	0	**52**	24	43	52	49	34	12
3	1	2	3	3	2	1	**53**	25	44	53	50	35	13
4	2	3	4	4	3	1	**54**	25	44	54	50	36	13
5	2	4	5	5	3	1	**55**	26	45	55	51	36	13
6	3	5	6	6	4	1	**56**	26	46	56	52	37	13
7	3	6	7	7	5	2	**57**	27	47	57	53	38	14
8	4	7	8	7	5	2	**58**	27	48	58	54	38	14
9	4	7	9	8	6	2	**59**	27	49	59	55	39	14
10	5	8	10	9	7	2	**60**	28	49	60	56	40	14
11	5	9	11	10	7	3	**61**	28	50	61	57	40	15
12	6	10	12	11	8	3	**62**	29	51	62	58	41	15
13	6	11	13	12	9	3	**63**	29	52	63	59	42	15
14	7	12	14	13	9	3	**64**	30	53	64	60	42	15
15	7	12	15	14	10	4	**65**	30	53	65	61	43	16
16	7	13	16	15	11	4	**66**	31	54	66	62	44	16
17	8	14	17	16	11	4	**67**	31	55	67	63	44	16
18	8	15	18	17	12	4	**68**	32	56	68	64	45	16
19	9	16	19	18	13	5	**69**	32	57	69	65	46	16
20	9	16	20	19	13	5	**70**	33	58	70	65	46	17
21	10	17	21	20	14	5	**71**	33	58	71	66	47	17
22	10	18	22	21	15	5	**72**	33	59	71	67	48	17
23	11	19	23	22	15	5	**73**	34	60	72	68	48	17
24	11	20	24	22	16	6	**74**	34	61	73	69	49	18
25	12	21	25	23	17	6	**75**	35	62	74	70	50	18
26	12	21	26	24	17	6	**76**	35	63	75	71	50	18
27	13	22	27	25	18	6	**77**	36	63	76	72	51	18
28	13	23	28	26	19	7	**78**	36	64	77	73	52	19
29	13	24	29	27	19	7	**79**	37	65	78	74	52	19
30	14	25	30	28	20	7	**80**	37	66	79	75	53	19
31	14	26	31	29	21	7	**81**	38	67	80	76	54	19
32	15	26	32	30	21	8	**82**	38	67	81	77	54	20
33	15	27	33	31	22	8	**83**	39	68	82	78	55	20
34	16	28	34	32	23	8	**84**	39	69	83	79	56	20
35	16	29	35	33	23	8	**85**	40	70	84	79	56	20
36	17	30	36	34	24	9	**86**	40	71	85	80	57	21
37	17	30	37	35	25	9	**87**	40	72	86	81	58	21
38	18	31	38	36	25	9	**88**	41	72	87	82	58	21
39	18	32	39	36	26	9	**89**	41	73	88	83	59	21
40	19	33	40	37	27	10	**90**	42	74	89	84	60	22
41	19	34	41	38	27	10	**91**	42	75	90	85	60	22
42	20	35	42	39	28	10	**92**	43	76	91	86	61	22
43	20	35	43	40	29	10	**93**	43	77	92	87	62	22
44	20	36	44	41	29	11	**94**	44	77	93	88	62	22
45	21	37	45	42	30	11	**95**	44	78	94	89	63	23
46	21	38	46	43	30	11	**96**	45	79	95	90	64	23
47	22	39	47	44	31	11	**97**	45	80	96	91	64	23
48	22	40	48	45	32	11	**98**	46	81	97	92	65	23
49	23	40	49	46	32	12	**99**	46	81	98	93	66	24
50	23	41	50	47	33	12	**100**	46	82	99	94	66	24
—	13	12	11	10	9	8	—	13	12	11	10	9	8

13 TERMS — COS θ

1 *is* 1·000

	+2	+3	+4	−5	−6	−7
Nos.	·886	·568	·120	·355	·748	·971
1	1	1	0	0	1	1
2	2	1	0	1	1	2
3	3	2	0	1	2	3
4	4	2	0	1	3	4
5	4	3	1	2	4	5
6	5	3	1	2	4	6
7	6	4	1	2	5	7
8	7	5	1	3	6	8
9	8	5	1	3	7	9
10	9	6	1	4	7	10
11	10	6	1	4	8	11
12	11	7	1	4	9	12
13	12	7	2	5	10	13
14	12	8	2	5	10	14
15	13	9	2	5	11	15
16	14	9	2	6	12	16
17	15	10	2	6	13	17
18	16	10	2	6	13	17
19	17	11	2	7	14	18
20	18	11	2	7	15	19
21	19	12	3	7	16	20
22	19	12	3	8	16	21
23	20	13	3	8	17	22
24	21	14	3	9	18	23
25	22	14	3	9	19	24
26	23	15	3	9	19	25
27	24	15	3	10	20	26
28	25	16	3	10	21	27
29	26	16	3	10	22	28
30	27	17	4	11	22	29
31	28	18	4	11	23	30
32	28	18	4	11	24	31
33	29	19	4	12	25	32
34	30	19	4	12	25	33
35	31	20	4	12	26	34
36	32	20	4	13	27	35
37	33	21	4	13	28	36
38	34	22	5	13	28	37
39	35	22	5	14	29	38
40	35	23	5	14	30	39
41	36	23	5	15	31	40
42	37	24	5	15	31	41
43	38	24	5	15	32	42
44	39	25	5	16	33	43
45	40	26	5	16	34	44
46	41	26	6	16	34	45
47	42	27	6	17	35	46
48	43	27	6	17	36	47
49	43	28	6	17	37	48
50	44	28	6	18	37	49
51	45	29	6	18	38	50
52	46	30	6	18	39	50
53	47	30	6	19	40	51
54	48	31	6	19	40	52
55	49	31	7	20	41	53
56	50	32	7	20	42	54
57	51	32	7	20	43	55
58	51	33	7	21	43	56
59	52	34	7	21	44	57
60	53	34	7	21	45	58
61	54	35	7	22	46	59
62	55	35	7	22	46	60
63	56	36	8	22	47	61
64	57	36	8	23	48	62
65	58	37	8	23	49	63
66	58	37	8	23	49	64
67	59	38	8	24	50	65
68	60	39	8	24	51	66
69	61	39	8	24	52	67
70	62	40	8	25	52	68
71	63	40	9	25	53	69
72	64	41	9	26	54	70
73	65	41	9	26	55	71
74	66	42	9	26	55	72
75	66	43	9	27	56	73
76	67	43	9	27	57	74
77	68	44	9	27	58	75
78	69	44	9	28	58	76
79	70	45	9	28	59	77
80	71	45	10	28	60	78
81	72	46	10	29	61	79
82	73	47	10	29	61	80
83	74	47	10	29	62	81
84	74	48	10	30	63	82
85	75	48	10	30	64	83
86	76	49	10	31	64	84
87	77	49	10	31	65	84
88	78	50	11	31	66	85
89	79	51	11	32	67	86
90	80	51	11	32	67	87
91	81	52	11	32	68	88
92	82	52	11	33	69	89
93	82	53	11	33	70	90
94	83	53	11	33	70	91
95	84	54	11	34	71	92
96	85	55	12	34	72	93
97	86	55	12	34	73	94
98	87	56	12	35	73	95
99	88	56	12	35	74	96
100	89	57	12	36	75	97
	+13	**+12**	**+11**	**−10**	**−9**	**−8**

14 TERMS — SINE θ

1 *and* **8** *are zero*

+	2 & 7	3 & 6	4 & 5
Nos.	·434	·782	·975
1	0	1	1
2	1	2	2
3	1	2	3
4	2	3	4
5	2	4	5
6	3	5	6
7	3	5	7
8	3	6	8
9	4	7	9
10	4	8	10
11	5	9	11
12	5	9	12
13	6	10	13
14	6	11	14
15	7	12	15
16	7	13	16
17	7	13	17
18	8	14	18
19	8	15	19
20	9	16	20
21	9	16	20
22	10	17	21
23	10	18	22
24	10	19	23
25	11	20	24
26	11	20	25
27	12	21	26
28	12	22	27
29	13	23	28
30	13	23	29
31	13	24	30
32	14	25	31
33	14	26	32
34	15	27	33
35	15	27	34
36	16	28	35
37	16	29	36
38	16	30	37
39	17	30	38
40	17	31	39
41	18	32	40
42	18	33	41
43	19	34	42
44	19	34	43
45	20	35	44
46	20	36	45
47	20	37	46
48	21	38	47
49	21	38	48
50	22	39	49
51	22	40	50
52	23	41	51
53	23	41	52
54	23	42	53
55	24	43	54
56	24	44	55
57	25	45	56
58	25	45	57
59	26	46	58
60	26	47	59
61	26	48	59
62	27	48	60
63	27	49	61
64	28	50	62
65	28	51	63
66	29	52	64
67	29	52	65
68	30	53	66
69	30	54	67
70	30	55	68
71	31	56	69
72	31	56	70
73	32	57	71
74	32	58	72
75	33	59	73
76	33	59	74
77	33	60	75
78	34	61	76
79	34	62	77
80	35	63	78
81	35	63	79
82	36	64	80
83	36	65	81
84	36	66	82
85	37	66	83
86	37	67	84
87	38	68	85
88	38	69	86
89	39	70	87
90	39	70	88
91	39	71	89
92	40	72	90
93	40	73	91
94	41	74	92
95	41	74	93
96	42	75	94
97	42	76	95
98	43	77	96
99	43	77	97
100	43	78	98
—	**9 & 14**	**10 & 13**	**11 & 12**

1 *is* + 1·000
8 *is* − 1·000

14 TERMS — COS θ

+	2 & 14	3 & 13	4 & 12
Nos.	·901	·623	·223
1	1	1	0
2	2	1	0
3	3	2	1
4	4	2	1
5	5	3	1
6	5	4	1
7	6	4	2
8	7	5	2
9	8	6	2
10	9	6	2
11	10	7	2
12	11	7	3
13	12	8	3
14	13	9	3
15	14	9	3
16	14	10	4
17	15	11	4
18	16	11	4
19	17	12	4
20	18	12	4
21	19	13	5
22	20	14	5
23	21	14	5
24	22	15	5
25	23	16	6
26	23	16	6
27	24	17	6
28	25	17	6
29	26	18	6
30	27	19	7
31	28	19	7
32	29	20	7
33	30	21	7
34	31	21	8
35	32	22	8
36	32	22	8
37	33	23	8
38	34	24	8
39	35	24	9
40	36	25	9
41	37	26	9
42	38	26	9
43	39	27	10
44	40	27	10
45	41	28	10
46	41	29	10
47	42	29	10
48	43	30	11
49	44	31	11
50	45	31	11
51	46	32	11
52	47	32	12
53	48	33	12
54	49	34	12
55	50	34	12
56	50	35	12
57	51	36	13
58	52	36	13
59	53	37	13
60	54	37	13
61	55	38	14
62	56	39	14
63	57	39	14
64	58	40	14
65	59	40	14
66	59	41	15
67	60	42	15
68	61	42	15
69	62	43	15
70	63	44	16
71	64	44	16
72	65	45	16
73	66	45	16
74	67	46	17
75	68	47	17
76	68	47	17
77	69	48	17
78	70	49	17
79	71	49	18
80	72	50	18
81	73	50	18
82	74	51	18
83	75	52	19
84	76	52	19
85	77	53	19
86	77	54	19
87	78	54	19
88	79	55	20
89	80	55	20
90	81	56	20
91	82	57	20
92	83	57	21
93	84	58	21
94	85	59	21
95	86	59	21
96	86	60	21
97	87	60	22
98	88	61	22
99	89	62	22
100	90	62	22
−	7 & 9	6 & 10	5 & 11

15 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6	7	8	+	2	3	4	5	6	7	8
Nos.	·407	·743	·951	·995	·866	·588	·208	Nos.	·407	·743	·951	·995	·866	·588	·208
1	0	1	1	1	1	1	0	**51**	21	38	49	51	44	30	11
2	1	1	2	2	2	1	0	**52**	21	39	49	52	45	31	11
3	1	2	3	3	3	2	1	**53**	22	39	50	53	46	31	11
4	2	3	4	4	3	2	1	**54**	22	40	51	54	47	32	11
5	2	4	5	5	4	3	1	**55**	22	41	52	55	48	32	11
6	2	4	6	6	5	4	1	**56**	23	42	53	56	48	33	12
7	3	5	7	7	6	4	1	**57**	23	42	54	57	49	34	12
8	3	6	8	8	7	5	2	**58**	24	43	55	58	50	34	12
9	4	7	9	9	8	5	2	**59**	24	44	56	59	51	35	12
10	4	7	10	10	9	6	2	**60**	24	45	57	60	52	35	12
11	4	8	10	11	10	6	2	**61**	25	45	58	61	53	36	13
12	5	9	11	12	10	7	2	**62**	25	46	59	62	54	36	13
13	5	10	12	13	11	8	3	**63**	26	47	60	63	55	37	13
14	6	10	13	14	12	8	3	**64**	26	48	61	64	55	38	13
15	6	11	14	15	13	9	3	**65**	26	48	62	65	56	38	14
16	7	12	15	16	14	9	3	**66**	27	49	63	66	57	39	14
17	7	13	16	17	15	10	4	**67**	27	50	64	67	58	39	14
18	7	13	17	18	16	11	4	**68**	28	51	65	68	59	40	14
19	8	14	18	19	16	11	4	**69**	28	51	66	69	60	41	14
20	8	15	19	20	17	12	4	**70**	28	52	67	70	61	41	15
21	9	16	20	21	18	12	4	**71**	29	53	68	71	61	42	15
22	9	16	21	22	19	13	5	**72**	29	53	68	72	62	42	15
23	9	17	22	23	20	14	5	**73**	30	54	69	73	63	43	15
24	10	18	23	24	21	14	5	**74**	30	55	70	74	64	44	15
25	10	19	24	25	22	15	5	**75**	31	56	71	75	65	44	16
26	11	19	25	26	23	15	5	**76**	31	56	72	76	66	45	16
27	11	20	26	27	23	16	6	**77**	31	57	73	77	67	45	16
28	11	21	27	28	24	16	6	**78**	32	58	74	78	68	46	16
29	12	22	28	29	25	17	6	**79**	32	59	75	79	68	46	16
30	12	22	29	30	26	18	6	**80**	33	59	76	80	69	47	17
31	13	23	29	31	27	18	6	**81**	33	60	77	81	70	48	17
32	13	24	30	32	28	19	7	**82**	33	61	78	82	71	48	17
33	13	25	31	33	29	19	7	**83**	34	62	79	83	72	49	17
34	14	25	32	34	29	20	7	**84**	34	62	80	84	73	49	17
35	14	26	33	35	30	21	7	**85**	35	63	81	85	74	50	18
36	15	27	34	36	31	21	7	**86**	35	64	82	86	74	51	18
37	15	27	35	37	32	22	8	**87**	35	65	83	87	75	51	18
38	15	28	36	38	33	22	8	**88**	36	65	84	88	76	52	18
39	16	29	37	39	34	23	8	**89**	36	66	85	89	77	52	19
40	16	30	38	40	35	24	8	**90**	37	67	86	90	78	53	19
41	17	30	39	41	36	24	9	**91**	37	68	87	91	79	54	19
42	17	31	40	42	36	25	9	**92**	37	68	87	92	80	54	19
43	18	32	41	43	37	25	9	**93**	38	69	88	93	81	55	19
44	18	33	42	44	38	26	9	**94**	38	70	89	94	81	55	20
45	18	33	43	45	39	26	9	**95**	39	71	90	95	82	56	20
46	19	34	44	46	40	27	10	**96**	39	71	91	96	83	56	20
47	19	35	45	47	41	28	10	**97**	39	72	92	97	84	57	20
48	20	36	46	48	42	28	10	**98**	40	73	93	98	85	58	20
49	20	36	47	49	42	29	10	**99**	40	74	94	99	86	58	21
50	20	37	48	50	43	29	10	**100**	41	74	95	100	87	59	21
—	**15**	**14**	**13**	**12**	**11**	**10**	**9**	—	**15**	**14**	**13**	**12**	**11**	**10**	**9**

15 TERMS — COS θ

1 *is* + 1·000

	+2	+3	+4	−5	−6	−7	−8		+2	+3	+4	−5	−6	−7	−8
Nos.	·914	·669	·309	·105	·500	·809	·978	Nos.	·914	·669	·309	·105	·500	·809	·978
1	1	1	0	0	1	1	1	**51**	47	34	16	5	26	41	50
2	2	1	1	0	1	2	2	**52**	48	35	16	5	26	42	51
3	3	2	1	0	2	2	3	**53**	48	35	16	6	27	43	52
4	4	3	1	0	2	3	4	**54**	49	36	17	6	27	44	53
5	5	3	2	1	3	4	5	**55**	50	37	17	6	28	44	54
6	5	4	2	1	3	5	6	**56**	51	37	17	6	28	45	55
7	6	5	2	1	4	6	7	**57**	52	38	18	6	29	46	56
8	7	5	2	1	4	6	8	**58**	53	39	18	6	29	47	57
9	8	6	3	1	5	7	9	**59**	54	39	18	6	30	48	58
10	9	7	3	1	5	8	10	**60**	55	40	19	6	30	49	59
11	10	7	3	1	6	9	11	**61**	56	41	19	6	31	49	60
12	11	8	4	1	6	10	12	**62**	57	41	19	7	31	50	61
13	12	9	4	1	7	11	13	**63**	58	42	19	7	32	51	62
14	13	9	4	1	7	11	14	**64**	58	43	20	7	32	52	63
15	14	10	5	2	8	12	15	**65**	59	43	20	7	33	53	64
16	15	11	5	2	8	13	16	**66**	60	44	20	7	33	53	65
17	16	11	5	2	9	14	17	**67**	61	45	21	7	34	54	66
18	16	12	6	2	9	15	18	**68**	62	45	21	7	34	55	67
19	17	13	6	2	10	15	19	**69**	63	46	21	7	35	56	67
20	18	13	6	2	10	16	20	**70**	64	47	22	7	35	57	68
21	19	14	6	2	11	17	21	**71**	65	47	22	7	36	57	69
22	20	15	7	2	11	18	22	**72**	66	48	22	8	36	58	70
23	21	15	7	2	12	19	22	**73**	67	49	23	8	37	59	71
24	22	16	7	3	12	19	23	**74**	68	50	23	8	37	60	72
25	23	17	8	3	13	20	24	**75**	69	50	23	8	38	61	73
26	24	17	8	3	13	21	25	**76**	69	51	23	8	38	61	74
27	25	18	8	3	14	22	26	**77**	70	52	24	8	39	62	75
28	26	19	9	3	14	23	27	**78**	71	52	24	8	39	63	76
29	27	19	9	3	15	23	28	**79**	72	53	24	8	40	64	77
30	27	20	9	3	15	24	29	**80**	73	54	25	8	40	65	78
31	28	21	10	3	16	25	30	**81**	74	54	25	9	41	66	79
32	29	21	10	3	16	26	31	**82**	75	55	25	9	41	66	80
33	30	22	10	3	17	27	32	**83**	76	56	26	9	42	67	81
34	31	23	11	4	17	28	33	**84**	77	56	26	9	42	68	82
35	32	23	11	4	18	28	34	**85**	78	57	26	9	43	69	83
36	33	24	11	4	18	29	35	**86**	79	58	27	9	43	70	84
37	34	25	11	4	19	30	36	**87**	80	58	27	9	44	70	85
38	35	25	12	4	19	31	37	**88**	80	59	27	9	44	71	86
39	36	26	12	4	20	32	38	**89**	81	60	28	9	45	72	87
40	37	27	12	4	20	32	39	**90**	82	60	28	9	45	73	88
41	37	27	13	4	21	33	40	**91**	83	61	28	10	46	74	89
42	38	28	13	4	21	34	41	**92**	84	62	28	10	46	74	90
43	39	29	13	5	22	35	42	**93**	85	62	29	10	47	75	91
44	40	29	14	5	22	36	43	**94**	86	63	29	10	47	76	92
45	41	30	14	5	23	36	44	**95**	87	64	29	10	48	77	93
46	42	31	14	5	23	37	45	**96**	88	64	30	10	48	78	94
47	43	31	15	5	24	38	46	**97**	89	65	30	10	49	78	95
48	44	32	15	5	24	39	47	**98**	90	66	30	10	49	79	96
49	45	33	15	5	25	40	48	**99**	90	66	31	10	50	80	97
50	46	33	15	5	25	40	49	**100**	91	67	31	11	50	81	98
	+15	**+14**	**+13**	**−12**	**−11**	**−10**	**−9**		**+15**	**+14**	**+13**	**−12**	**−11**	**−10**	**−9**

1 *and* **9** *are zero* 16 **TERMS — SINE** *θ*
5 *and* **13** *are* 1·000

+	2 & 8	3 & 7	4 & 6
Nos.	·383	·707	·924
1	0	1	1
2	1	1	2
3	1	2	3
4	2	3	4
5	2	4	5
6	2	4	6
7	3	5	6
8	3	6	7
9	3	6	8
10	4	7	9
11	4	8	10
12	5	8	11
13	5	9	12
14	5	10	13
15	6	11	14
16	6	11	15
17	7	12	16
18	7	13	17
19	7	13	18
20	8	14	18
21	8	15	19
22	8	16	20
23	9	16	21
24	9	17	22
25	10	18	23
26	10	18	24
27	10	19	25
28	11	20	26
29	11	21	27
30	11	21	28
31	12	22	29
32	12	23	30
33	13	23	30
34	13	24	31
35	13	25	32
36	14	25	33
37	14	26	34
38	15	27	35
39	15	28	36
40	15	28	37
41	16	29	38
42	16	30	39
43	16	30	40
44	17	31	41
45	17	32	42
46	18	33	43
47	18	33	43
48	18	34	44
49	19	35	45
50	19	35	46
51	20	36	47
52	20	37	48
53	20	37	49
54	21	38	50
55	21	39	51
56	21	40	52
57	22	40	53
58	22	41	54
59	23	42	55
60	23	42	55
61	23	43	56
62	24	44	57
63	24	45	58
64	25	45	59
65	25	46	60
66	25	47	61
67	26	47	62
68	26	48	63
69	26	49	64
70	27	49	65
71	27	50	66
72	28	51	67
73	28	52	67
74	28	52	68
75	29	53	69
76	29	54	70
77	29	54	71
78	30	55	72
79	30	56	73
80	31	57	74
81	31	57	75
82	31	58	76
83	32	59	77
84	32	59	78
85	33	60	79
86	33	61	79
87	33	62	80
88	34	62	81
89	34	63	82
90	34	64	83
91	35	64	84
92	35	65	85
93	36	66	86
94	36	66	87
95	36	67	88
96	37	68	89
97	37	69	90
98	38	69	91
99	38	70	91
100	38	71	92
—	**10 & 16**	**11 & 15**	**12 & 14**

1 *and* **9** *are* 1·000   16 TERMS — COS θ
5 *and* **13** *are zero*

+	**2 & 16**	**3 & 15**	**4 & 14**
Nos.	·924	·707	·383
1	1	1	0
2	2	1	1
3	3	2	1
4	4	3	2
5	5	4	2
6	6	4	2
7	6	5	3
8	7	6	3
9	8	6	3
10	9	7	4
11	10	8	4
12	11	8	5
13	12	9	5
14	13	10	5
15	14	11	6
16	15	11	6
17	16	12	7
18	17	13	7
19	18	13	7
20	18	14	8
21	19	15	8
22	20	16	8
23	21	16	9
24	22	17	9
25	23	18	10
26	24	18	10
27	25	19	10
28	26	20	11
29	27	21	11
30	28	21	11
31	29	22	12
32	30	23	12
33	30	23	13
34	31	24	13
35	32	25	13
36	33	25	14
37	34	26	14
38	35	27	15
39	36	28	15
40	37	28	15
41	38	29	16
42	39	30	16
43	40	30	16
44	41	31	17
45	42	32	17
46	43	33	18
47	43	33	18
48	44	34	18
49	45	35	19
50	46	35	19
51	47	36	20
52	48	37	20
53	49	37	20
54	50	38	21
55	51	39	21
56	52	40	21
57	53	40	22
58	54	41	22
59	55	42	23
60	55	42	23
61	56	43	23
62	57	44	24
63	58	45	24
64	59	45	25
65	60	46	25
66	61	47	25
67	62	47	26
68	63	48	26
69	64	49	26
70	65	49	27
71	66	50	27
72	67	51	28
73	67	52	28
74	68	52	28
75	69	53	29
76	70	54	29
77	71	54	29
78	72	55	30
79	73	56	30
80	74	57	31
81	75	57	31
82	76	58	31
83	77	59	32
84	78	59	32
85	79	60	33
86	79	61	33
87	80	62	33
88	81	62	34
89	82	63	34
90	83	64	34
91	84	64	35
92	85	65	35
93	86	66	36
94	87	66	36
95	88	67	36
96	89	68	37
97	90	69	37
98	91	69	38
99	91	70	38
100	92	71	38
—	**10 & 8**	**11 & 7**	**12 & 6**

17 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6	7	8	9
Nos.	·361	·674	·895	·996	·962	·798	·527	·184
1	0	1	1	1	1	1	1	0
2	1	1	2	2	2	2	1	0
3	1	2	3	3	3	2	2	1
4	1	3	4	4	4	3	2	1
5	2	3	4	5	5	4	3	1
6	2	4	5	6	6	5	3	1
7	3	5	6	7	7	6	4	1
8	3	5	7	8	8	6	4	1
9	3	6	8	9	9	7	5	2
10	4	7	9	10	10	8	5	2
11	4	7	10	11	11	9	6	2
12	4	8	11	12	12	10	6	2
13	5	9	12	13	13	10	7	2
14	5	9	13	14	13	11	7	3
15	5	10	13	15	14	12	8	3
16	6	11	14	16	15	13	8	3
17	6	11	15	17	16	14	9	3
18	6	12	16	18	17	14	9	3
19	7	13	17	19	18	15	10	3
20	7	13	18	20	19	16	11	4
21	8	14	19	21	20	17	11	4
22	8	15	20	22	21	18	12	4
23	8	16	21	23	22	18	12	4
24	9	16	21	24	23	19	13	4
25	9	17	22	25	24	20	13	5
26	9	18	23	26	25	21	14	5
27	10	18	24	27	26	22	14	5
28	10	19	25	28	27	22	15	5
29	10	20	26	29	28	23	15	5
30	11	20	27	30	29	24	16	6
31	11	21	28	31	30	25	16	6
32	12	22	29	32	31	26	17	6
33	12	22	30	33	32	26	17	6
34	12	23	30	34	33	27	18	6
35	13	24	31	35	34	28	18	6
36	13	24	32	36	35	29	19	7
37	13	25	33	37	36	30	19	7
38	14	26	34	38	37	30	20	7
39	14	26	35	39	38	31	21	7
40	14	27	36	40	38	32	21	7
41	15	28	37	41	39	33	22	8
42	15	28	38	42	40	34	22	8
43	16	29	38	43	41	34	23	8
44	16	30	39	44	42	35	23	8
45	16	30	40	45	43	36	24	8
46	17	31	41	46	44	37	24	8
47	17	32	42	47	45	38	25	9
48	17	32	43	48	46	38	25	9
49	18	33	44	49	47	39	26	9
50	18	34	45	50	48	40	26	9
—	**17**	**16**	**15**	**14**	**13**	**12**	**11**	**10**

17 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6	7	8	9
Nos.	·361	·674	·895	·996	·962	·798	·527	·184
51	18	34	46	51	49	41	27	9
52	19	35	47	52	50	41	27	10
53	19	36	47	53	51	42	28	10
54	19	36	48	54	52	43	28	10
55	20	37	49	55	53	44	29	10
56	20	38	50	56	54	45	30	10
57	21	38	51	57	55	45	30	10
58	21	39	52	58	56	46	31	11
59	21	40	53	59	57	47	31	11
60	22	40	54	60	58	48	32	11
61	22	41	55	61	59	49	32	11
62	22	42	55	62	60	49	33	11
63	23	42	56	63	61	50	33	12
64	23	43	57	64	62	51	34	12
65	23	44	58	65	63	52	34	12
66	24	44	59	66	63	53	35	12
67	24	45	60	67	64	53	35	12
68	24	46	61	68	65	54	36	13
69	25	47	62	69	66	55	36	13
70	25	47	63	70	67	56	37	13
71	26	48	64	71	68	57	37	13
72	26	49	64	72	69	57	38	13
73	26	49	65	73	70	58	38	13
74	27	50	66	74	71	59	39	14
75	27	51	67	75	72	60	40	14
76	27	51	68	76	73	61	40	14
77	28	52	69	77	74	61	41	14
78	28	53	70	78	75	62	41	14
79	28	53	71	79	76	63	42	15
80	29	54	72	80	77	64	42	15
81	29	55	72	81	78	65	43	15
82	30	55	73	82	79	65	43	15
83	30	56	74	83	80	66	44	15
84	30	57	75	84	81	67	44	15
85	31	57	76	85	82	68	45	16
86	31	58	77	86	83	69	45	16
87	31	59	78	87	84	69	46	16
88	32	59	79	88	85	70	46	16
89	32	60	80	89	86	71	47	16
90	32	61	81	90	87	72	47	17
91	33	61	81	91	88	73	48	17
92	33	62	82	92	89	73	48	17
93	33	63	83	93	89	74	49	17
94	34	63	84	94	90	75	50	17
95	34	64	85	95	91	76	50	17
96	35	65	86	96	92	77	51	18
97	35	65	87	97	93	77	51	18
98	35	66	88	98	94	78	52	18
99	36	67	89	99	95	79	52	18
100	36	67	90	100	96	80	53	18
—	**17**	**16**	**15**	**14**	**13**	**12**	**11**	**10**

17 TERMS — COS θ

1 *is* 1·000

	+2	+3	+4	+5	−6	−7	−8	−9
Nos.	·933	·739	·445	·092	·274	·602	·850	·983
1	1	1	0	0	0	1	1	1
2	2	1	1	0	1	1	2	2
3	3	2	1	0	1	2	3	3
4	4	3	2	0	1	2	3	4
5	5	4	2	0	1	3	4	5
6	6	4	3	1	2	4	5	6
7	7	5	3	1	2	4	6	7
8	7	6	4	1	2	5	7	8
9	8	7	4	1	2	5	8	9
10	9	7	4	1	3	6	9	10
11	10	8	5	1	3	7	9	11
12	11	9	5	1	3	7	10	12
13	12	10	6	1	4	8	11	13
14	13	10	6	1	4	8	12	14
15	14	11	7	1	4	9	13	15
16	15	12	7	1	4	10	14	16
17	16	13	8	2	5	10	14	17
18	17	13	8	2	5	11	15	18
19	18	14	8	2	5	11	16	19
20	19	15	9	2	5	12	17	20
21	20	16	9	2	6	13	18	21
22	21	16	10	2	6	13	19	22
23	21	17	10	2	6	14	20	23
24	22	18	11	2	7	14	20	24
25	23	18	11	2	7	15	21	25
26	24	19	12	2	7	16	22	26
27	25	20	12	2	7	16	23	27
28	26	21	12	3	8	17	24	28
29	27	21	13	3	8	17	25	29
30	28	22	13	3	8	18	26	29
31	29	23	14	3	8	19	26	30
32	30	24	14	3	9	19	27	31
33	31	24	15	3	9	20	28	32
34	32	25	15	3	9	20	29	33
35	33	26	16	3	10	21	30	34
36	34	27	16	3	10	22	31	35
37	35	27	16	3	10	22	31	36
38	35	28	17	3	10	23	32	37
39	36	29	17	4	11	23	33	38
40	37	30	18	4	11	24	34	39
41	38	30	18	4	11	25	35	40
42	39	31	19	4	12	25	36	41
43	40	32	19	4	12	26	37	42
44	41	33	20	4	12	26	37	43
45	42	33	20	4	12	27	38	44
46	43	34	20	4	13	28	39	45
47	44	35	21	4	13	28	40	46
48	45	35	21	4	13	29	41	47
49	46	36	22	5	13	29	42	48
50	47	37	22	5	14	30	43	49
	+17	+16	+15	+14	−13	−12	−11	−10

1 *is* 1·000

	+2	+3	+4	+5	−6	−7	−8	−9
Nos.	·933	·739	·445	·092	·274	·602	·850	·983
51	48	38	23	5	14	31	43	50
52	49	38	23	5	14	31	44	51
53	49	39	24	5	15	32	45	52
54	50	40	24	5	15	33	46	53
55	51	41	24	5	15	33	47	54
56	52	41	25	5	15	34	48	55
57	53	42	25	5	16	34	48	56
58	54	43	26	5	16	35	49	57
59	55	44	26	5	16	36	50	58
60	56	44	27	6	16	36	51	59
61	57	45	27	6	17	37	52	60
62	58	46	28	6	17	37	53	61
63	59	47	28	6	17	38	54	62
64	60	47	28	6	18	39	54	63
65	61	48	29	6	18	39	55	64
66	62	49	29	6	18	40	56	65
67	63	50	30	6	18	40	57	66
68	63	50	30	6	19	41	58	67
69	64	51	31	6	19	42	59	68
70	65	52	31	6	19	42	60	69
71	66	52	32	7	19	43	60	70
72	67	53	32	7	20	43	61	71
73	68	54	32	7	20	44	62	72
74	69	55	33	7	20	45	63	73
75	70	55	33	7	21	45	64	74
76	71	56	34	7	21	46	65	75
77	72	57	34	7	21	46	65	76
78	73	58	35	7	21	47	66	77
79	74	58	35	7	22	48	67	78
80	75	59	36	7	22	48	68	79
81	76	60	36	7	22	49	69	80
82	77	61	36	8	22	49	70	81
83	77	61	37	8	23	50	71	82
84	78	62	37	8	23	51	71	83
85	79	63	38	8	23	51	72	84
86	80	64	38	8	24	52	73	85
87	81	64	39	8	24	52	74	86
88	82	65	39	8	24	53	75	87
89	83	66	40	8	24	54	76	87
90	84	67	40	8	25	54	77	88
91	85	67	40	8	25	55	77	89
92	86	68	41	8	25	55	78	90
93	87	69	41	9	25	56	79	91
94	88	69	42	9	26	57	80	92
95	89	70	42	9	26	57	81	93
96	90	71	43	9	26	58	82	94
97	91	72	43	9	27	58	82	95
98	91	72	44	9	27	59	83	96
99	92	73	44	9	27	60	84	97
100	93	74	45	9	27	60	85	98
	+17	+16	+15	+14	−13	−12	−11	−10

18 TERMS — SINE θ

1 *and* **10** *are zero*

+	2 & 9	3 & 8	4 & 7	5 & 6
Nos.	·342	·643	·866	·985
1	0	1	1	1
2	1	1	2	2
3	1	2	3	3
4	1	3	3	4
5	2	3	4	5
6	2	4	5	6
7	2	5	6	7
8	3	5	7	8
9	3	6	8	9
10	3	6	9	10
11	4	7	10	11
12	4	8	10	12
13	4	8	11	13
14	5	9	12	14
15	5	10	13	15
16	5	10	14	16
17	6	11	15	17
18	6	12	16	18
19	6	12	16	19
20	7	13	17	20
21	7	14	18	21
22	8	14	19	22
23	8	15	20	23
24	8	15	21	24
25	9	16	22	25
26	9	17	23	26
27	9	17	23	27
28	10	18	24	28
29	10	19	25	29
30	10	19	26	30
31	11	20	27	31
32	11	21	28	32
33	11	21	29	33
34	12	22	29	33
35	12	23	30	34
36	12	23	31	35
37	13	24	32	36
38	13	24	33	37
39	13	25	34	38
40	14	26	35	39
41	14	26	36	40
42	14	27	36	41
43	15	28	37	42
44	15	28	38	43
45	15	29	39	44
46	16	30	40	45
47	16	30	41	46
48	16	31	42	47
49	17	32	42	48
50	17	32	43	49
51	17	33	44	50
52	18	33	45	51
53	18	34	46	52
54	18	35	47	53
55	19	35	48	54
56	19	36	48	55
57	19	37	49	56
58	20	37	50	57
59	20	38	51	58
60	21	39	52	59
61	21	39	53	60
62	21	40	54	61
63	22	41	55	62
64	22	41	55	63
65	22	42	56	64
66	23	42	57	65
67	23	43	58	66
68	23	44	59	67
69	24	44	60	68
70	24	45	61	69
71	24	46	61	70
72	25	46	62	71
73	25	47	63	72
74	25	48	64	73
75	26	48	65	74
76	26	49	66	75
77	26	50	67	76
78	27	50	68	77
79	27	51	68	78
80	27	51	69	79
81	28	52	70	80
82	28	53	71	81
83	28	53	72	82
84	29	54	73	83
85	29	55	74	84
86	29	55	74	85
87	30	56	75	86
88	30	57	76	87
89	30	57	77	88
90	31	58	78	89
91	31	59	79	90
92	31	59	80	91
93	32	60	81	92
94	32	60	81	93
95	32	61	82	94
96	33	62	83	95
97	33	62	84	96
98	34	63	85	97
99	34	64	86	98
100	34	64	87	99
—	**11 & 18**	**12 & 17**	**13 & 16**	**14 & 15**

1 *and* 10 *are* 1·000

+	2 & 18	3 & 17	4 & 16	5 & 15
Nos.	·940	·766	·500	·174
1	1	1	1	0
2	2	2	1	0
3	3	2	2	1
4	4	3	2	1
5	5	4	3	1
6	6	5	3	1
7	7	5	4	1
8	8	6	4	1
9	8	7	5	2
10	9	8	5	2
11	10	8	6	2
12	11	9	6	2
13	12	10	7	2
14	13	11	7	2
15	14	11	8	3
16	15	12	8	3
17	16	13	9	3
18	17	14	9	3
19	18	15	10	3
20	19	15	10	3
21	20	16	11	4
22	21	17	11	4
23	22	18	12	4
24	23	18	12	4
25	24	19	13	4
26	24	20	13	5
27	25	21	14	5
28	26	21	14	5
29	27	22	15	5
30	28	23	15	5
31	29	24	16	5
32	30	25	16	6
33	31	25	17	6
34	32	26	17	6
35	33	27	18	6
36	34	28	18	6
37	35	28	19	6
38	36	29	19	7
39	37	30	20	7
40	38	31	20	7
41	39	31	21	7
42	39	32	21	7
43	40	33	22	7
44	41	34	22	8
45	42	34	23	8
46	43	35	23	8
47	44	36	24	8
48	45	37	24	8
49	46	38	25	9
50	47	38	25	9
51	48	39	26	9
52	49	40	26	9
53	50	41	27	9
54	51	41	27	9
55	52	42	28	10
56	53	43	28	10
57	54	44	29	10
58	55	44	29	10
59	55	45	30	10
60	56	46	30	10
61	57	47	31	11
62	58	47	31	11
63	59	48	32	11
64	60	49	32	11
65	61	50	33	11
66	62	51	33	11
67	63	51	34	12
68	64	52	34	12
69	65	53	35	12
70	66	54	35	12
71	67	54	36	12
72	68	55	36	13
73	69	56	37	13
74	70	57	37	13
75	71	57	38	13
76	71	58	38	13
77	72	59	39	13
78	73	60	39	14
79	74	61	40	14
80	75	61	40	14
81	76	62	41	14
82	77	63	41	14
83	78	64	42	14
84	79	64	42	15
85	80	65	43	15
86	81	66	43	15
87	82	67	44	15
88	83	67	44	15
89	84	68	45	15
90	85	69	45	16
91	86	70	46	16
92	86	70	46	16
93	87	71	47	16
94	88	72	47	16
95	89	73	48	17
96	90	74	48	17
97	91	74	49	17
98	92	75	49	17
99	93	76	50	17
100	94	77	50	17
—	9 & 11	8 & 12	7 & 13	6 & 14

19 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6	7	8	9	10
Nos.	·325	·614	·837	·969	·997	·916	·736	·476	·164
1	0	1	1	1	1	1	1	0	0
2	1	1	2	2	2	2	1	1	0
3	1	2	3	3	3	3	2	1	0
4	1	2	3	4	4	4	3	2	1
5	2	3	4	5	5	5	4	2	1
6	2	4	5	6	6	5	4	3	1
7	2	4	6	7	7	6	5	3	1
8	3	5	7	8	8	7	6	4	1
9	3	6	8	9	9	8	7	4	1
10	3	6	8	10	10	9	7	5	2
11	4	7	9	11	11	10	8	5	2
12	4	7	10	12	12	11	9	6	2
13	4	8	11	13	13	12	10	6	2
14	5	9	12	14	14	13	10	7	2
15	5	9	13	15	15	14	11	7	2
16	5	10	13	16	16	15	12	8	3
17	6	10	14	16	17	16	13	8	3
18	6	11	15	17	18	16	13	9	3
19	6	12	16	18	19	17	14	9	3
20	7	12	17	19	20	18	15	10	3
21	7	13	18	20	21	19	15	10	3
22	7	14	18	21	22	20	16	10	4
23	7	14	19	22	23	21	17	11	4
24	8	15	20	23	24	22	18	11	4
25	8	15	21	24	25	23	18	12	4
26	8	16	22	25	26	24	19	12	4
27	9	17	23	26	27	25	20	13	4
28	9	17	23	27	28	26	21	13	5
29	9	18	24	28	29	27	21	14	5
30	10	18	25	29	30	27	22	14	5
31	10	19	26	30	31	28	23	15	5
32	10	20	27	31	32	29	24	15	5
33	11	20	28	32	33	30	24	16	5
34	11	21	28	33	34	31	25	16	6
35	11	21	29	34	35	32	26	17	6
36	12	22	30	35	36	33	26	17	6
37	12	23	31	36	37	34	27	18	6
38	12	23	32	37	38	35	28	18	6
39	13	24	33	38	39	36	29	19	6
40	13	25	33	39	40	37	29	19	7
41	13	25	34	40	41	38	30	20	7
42	14	26	35	41	42	38	31	20	7
43	14	26	36	42	43	39	32	20	7
44	14	27	37	43	44	40	32	21	7
45	15	28	38	44	45	41	33	21	7
46	15	28	39	45	46	42	34	22	8
47	15	29	39	46	47	43	35	22	8
48	16	29	40	47	48	44	35	23	8
49	16	30	41	47	49	45	36	23	8
50	16	31	42	48	50	46	37	24	8
—	**19**	**18**	**17**	**16**	**15**	**14**	**13**	**12**	**11**

19 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6	7	8	9	10
Nos.	·325	·614	·837	·969	·997	·916	·736	·476	·164
51	17	31	43	49	51	47	38	24	8
52	17	32	44	50	52	48	38	25	9
53	17	33	44	51	53	49	39	25	9
54	18	33	45	52	54	49	40	26	9
55	18	34	46	53	55	50	40	26	9
56	18	34	47	54	56	51	41	27	9
57	19	35	48	55	57	52	42	27	9
58	19	36	49	56	58	53	43	28	10
59	19	36	49	57	59	54	43	28	10
60	20	37	50	58	60	55	44	29	10
61	20	37	51	59	61	56	45	29	10
62	20	38	52	60	62	57	46	30	10
63	20	39	53	61	63	58	46	30	10
64	21	39	54	62	64	59	47	30	10
65	21	40	54	63	65	60	48	31	11
66	21	41	55	64	66	60	49	31	11
67	22	41	56	65	67	61	49	32	11
68	22	42	57	66	68	62	50	32	11
69	22	42	58	67	69	63	51	33	11
70	23	43	59	68	70	64	52	33	11
71	23	44	59	69	71	65	52	34	12
72	23	44	60	70	72	66	53	34	12
73	24	45	61	71	73	67	54	35	12
74	24	45	62	72	74	68	54	35	12
75	24	46	63	73	75	69	55	36	12
76	25	47	64	74	76	70	56	36	12
77	25	47	64	75	77	71	57	37	13
78	25	48	65	76	78	71	57	37	13
79	26	49	66	77	79	72	58	38	13
80	26	49	67	78	80	73	59	38	13
81	26	50	68	78	81	74	60	39	13
82	27	50	69	79	82	75	60	39	13
83	27	51	69	80	83	76	61	40	14
84	27	52	70	81	84	77	62	40	14
85	28	52	71	82	85	78	63	40	14
86	28	53	72	83	86	79	63	41	14
87	28	53	73	84	87	80	64	41	14
88	29	54	74	85	88	81	65	42	14
89	29	55	74	86	89	82	66	42	15
90	29	55	75	87	90	82	66	43	15
91	30	56	76	88	91	83	67	43	15
92	30	56	77	89	92	84	68	44	15
93	30	57	78	90	93	85	68	44	15
94	31	58	79	91	94	86	69	45	15
95	31	58	80	92	95	87	70	45	16
96	31	59	80	93	96	88	71	46	16
97	32	60	81	94	97	89	71	46	16
98	32	60	82	95	98	90	72	47	16
99	32	61	83	96	99	91	73	47	16
100	33	61	84	97	100	92	74	48	16
—	19	18	17	16	15	14	13	12	11

19 TERMS — COS θ

1 *is* + 1·000

	+2	+3	+4	+5	−6	−7	−8	−9	−10
Nos.	·946	·789	·547	·246	·082	·401	·677	·879	·986
1	1	1	1	0	0	0	1	1	1
2	2	2	1	0	0	1	1	2	2
3	3	2	2	1	0	1	2	3	3
4	4	3	2	1	0	2	3	4	4
5	5	4	3	1	0	2	3	4	5
6	6	5	3	1	0	2	4	5	6
7	7	6	4	2	1	3	5	6	7
8	8	6	4	2	1	3	5	7	8
9	9	7	5	2	1	4	6	8	9
10	9	8	5	2	1	4	7	9	10
11	10	9	6	3	1	4	7	10	11
12	11	9	7	3	1	5	8	11	12
13	12	10	7	3	1	5	9	11	13
14	13	11	8	3	1	6	9	12	14
15	14	12	8	4	1	6	10	13	15
16	15	13	9	4	1	6	11	14	16
17	16	13	9	4	1	7	12	15	17
18	17	14	10	4	1	7	12	16	18
19	18	15	10	5	2	8	13	17	19
20	19	16	11	5	2	8	14	18	20
21	20	17	11	5	2	8	14	18	21
22	21	17	12	5	2	9	15	19	22
23	22	18	13	6	2	9	16	20	23
24	23	19	13	6	2	10	16	21	24
25	24	20	14	6	2	10	17	22	25
26	25	21	14	6	2	10	18	23	26
27	26	21	15	7	2	11	18	24	27
28	26	22	15	7	2	11	19	25	28
29	27	23	16	7	2	12	20	25	29
30	28	24	16	7	2	12	20	26	30
31	29	24	17	8	3	12	21	27	31
32	30	25	18	8	3	13	22	28	32
33	31	26	18	8	3	13	22	29	33
34	32	27	19	8	3	14	23	30	34
35	33	28	19	9	3	14	24	31	35
36	34	28	20	9	3	14	24	32	35
37	35	29	20	9	3	15	25	33	36
38	36	30	21	9	3	15	26	33	37
39	37	31	21	10	3	16	26	34	38
40	38	32	22	10	3	16	27	35	39
41	39	32	22	10	3	16	28	36	40
42	40	33	23	10	3	17	28	37	41
43	41	34	24	11	4	17	29	38	42
44	42	35	24	11	4	18	30	39	43
45	43	36	25	11	4	18	30	40	44
46	44	36	25	11	4	18	31	40	45
47	44	37	26	12	4	19	32	41	46
48	45	38	26	12	4	19	32	42	47
49	46	39	27	12	4	20	33	43	48
50	47	39	27	12	4	20	34	44	49
	+19	+18	+17	+16	−15	−14	−13	−12	−11

19 TERMS — COS θ

1 *is* + 1·000

	+2	+3	+4	+5	−6	−7	−8	−9	−10
Nos.	·946	·789	·547	·246	·082	·401	·677	·879	·986
51	48	40	28	13	4	20	35	45	50
52	49	41	28	13	4	21	35	46	51
53	50	42	29	13	4	21	36	47	52
54	51	43	30	13	4	22	37	47	53
55	52	43	30	14	5	22	37	48	54
56	53	44	31	14	5	22	38	49	55
57	54	45	31	14	5	23	39	50	56
58	55	46	32	14	5	23	39	51	57
59	56	47	32	15	5	24	40	52	58
60	57	47	33	15	5	24	41	53	59
61	58	48	33	15	5	24	41	54	60
62	59	49	34	15	5	25	42	54	61
63	60	50	34	15	5	25	43	55	62
64	61	50	35	16	5	26	43	56	63
65	61	51	36	16	5	26	44	57	64
66	62	52	36	16	5	26	45	58	65
67	63	53	37	16	5	27	45	59	66
68	64	54	37	17	6	27	46	60	67
69	65	54	38	17	6	28	47	61	68
70	66	55	38	17	6	28	47	62	69
71	67	56	39	17	6	28	48	62	70
72	68	57	39	18	6	29	49	63	71
73	69	58	40	18	6	29	49	64	72
74	70	58	40	18	6	30	50	65	73
75	71	59	41	18	6	30	51	66	74
76	72	60	42	19	6	30	51	67	75
77	73	61	42	19	6	31	52	68	76
78	74	62	43	19	6	31	53	69	77
79	75	62	43	19	6	32	53	69	78
80	76	63	44	20	7	32	54	70	79
81	77	64	44	20	7	32	55	71	80
82	78	65	45	20	7	33	56	72	81
83	79	65	45	20	7	33	56	73	82
84	79	66	46	21	7	34	57	74	83
85	80	67	46	21	7	34	58	75	84
86	81	68	47	21	7	34	58	76	85
87	82	69	48	21	7	35	59	76	86
88	83	69	48	22	7	35	60	77	87
89	84	70	49	22	7	36	60	78	88
90	85	71	49	22	7	36	61	79	89
91	86	72	50	22	7	36	62	80	90
92	87	73	50	23	8	37	62	81	91
93	88	73	51	23	8	37	63	82	92
94	89	74	51	23	8	38	64	83	93
95	90	75	52	23	8	38	64	84	94
96	91	76	53	24	8	38	65	84	95
97	92	77	53	24	8	39	66	85	96
98	93	77	54	24	8	39	66	86	97
99	94	78	54	24	8	40	67	87	98
100	95	79	55	25	8	40	68	88	99
	+19	**+18**	**+17**	**+16**	**−15**	**−14**	**−13**	**−12**	**−11**

1 *and* **11** *are zero*
6 *and* **16** *are* 1·000

20 TERMS — SINE θ

+	2 & 10	3 & 9	4 & 8	5 & 7
Nos.	·309	·588	·809	·951
1	0	1	1	1
2	1	1	2	2
3	1	2	2	3
4	1	2	3	4
5	2	3	4	5
6	2	4	5	6
7	2	4	6	7
8	2	5	6	8
9	3	5	7	9
10	3	6	8	10
11	3	6	9	11
12	4	7	10	11
13	4	8	11	12
14	4	8	11	13
15	5	9	12	14
16	5	9	13	15
17	5	10	14	16
18	6	11	15	17
19	6	11	15	18
20	6	12	16	19
21	6	12	17	20
22	7	13	18	21
23	7	14	19	22
24	7	14	19	23
25	8	15	20	24
26	8	15	21	25
27	8	16	22	26
28	9	16	23	27
29	9	17	23	28
30	9	18	24	29
31	10	18	25	29
32	10	19	26	30
33	10	19	27	31
34	11	20	28	32
35	11	21	28	33
36	11	21	29	34
37	11	22	30	35
38	12	22	31	36
39	12	23	32	37
40	12	24	32	38
41	13	24	33	39
42	13	25	34	40
43	13	25	35	41
44	14	26	36	42
45	14	26	36	43
46	14	27	37	44
47	15	28	38	45
48	15	28	39	46
49	15	29	40	47
50	15	29	40	48
51	16	30	41	49
52	16	31	42	49
53	16	31	43	50
54	17	32	44	51
55	17	32	44	52
56	17	33	45	53
57	18	34	46	54
58	18	34	47	55
59	18	35	48	56
60	19	35	49	57
61	19	36	49	58
62	19	36	50	59
63	19	37	51	60
64	20	38	52	61
65	20	38	53	62
66	20	39	53	63
67	21	39	54	64
68	21	40	55	65
69	21	41	56	66
70	22	41	57	67
71	22	42	57	68
72	22	42	58	68
73	23	43	59	69
74	23	44	60	70
75	23	44	61	71
76	23	45	61	72
77	24	45	62	73
78	24	46	63	74
79	24	46	64	75
80	25	47	65	76
81	25	48	66	77
82	25	48	66	78
83	26	49	67	79
84	26	49	68	80
85	26	50	69	81
86	27	51	70	82
87	27	51	70	83
88	27	52	71	84
89	28	52	72	85
90	28	53	73	86
91	28	54	74	87
92	28	54	74	87
93	29	55	75	88
94	29	55	76	89
95	29	56	77	90
96	30	56	78	91
97	30	57	78	92
98	30	58	79	93
99	31	58	80	94
100	31	59	81	95
—	**12 & 20**	**13 & 19**	**14 & 18**	**15 & 17**

1 *and* **11** *are* 1·000 — 20 TERMS — COS θ

6 *and* **16** *are zero*

+	2 & 20	3 & 19	4 & 18	5 & 17
Nos.	·951	·809	·588	·309
1	1	1	1	0
2	2	2	1	1
3	3	2	2	1
4	4	3	2	1
5	5	4	3	2
6	6	5	4	2
7	7	6	4	2
8	8	6	5	2
9	9	7	5	3
10	10	8	6	3
11	11	9	6	3
12	11	10	7	4
13	12	11	8	4
14	13	11	8	4
15	14	12	9	5
16	15	13	9	5
17	16	14	10	5
18	17	15	11	6
19	18	15	11	6
20	19	16	12	6
21	20	17	12	6
22	21	18	13	7
23	22	19	14	7
24	23	19	14	7
25	24	20	15	8
26	25	21	15	8
27	26	22	16	8
28	27	23	16	9
29	28	23	17	9
30	29	24	18	9
31	29	25	18	10
32	30	26	19	10
33	31	27	19	10
34	32	28	20	11
35	33	28	21	11
36	34	29	21	11
37	35	30	22	11
38	36	31	22	12
39	37	32	23	12
40	38	32	24	12
41	39	33	24	13
42	40	34	25	13
43	41	35	25	13
44	42	36	26	14
45	43	36	26	14
46	44	37	27	14
47	45	38	28	15
48	46	39	28	15
49	47	40	29	15
50	48	40	29	15
51	49	41	30	16
52	49	42	31	16
53	50	43	31	16
54	51	44	32	17
55	52	44	32	17
56	53	45	33	17
57	54	46	34	18
58	55	47	34	18
59	56	48	35	18
60	57	49	35	19
61	58	49	36	19
62	59	50	36	19
63	60	51	37	19
64	61	52	38	20
65	62	53	38	20
66	63	53	39	20
67	64	54	39	21
68	65	55	40	21
69	66	56	41	21
70	67	57	41	22
71	68	57	42	22
72	68	58	42	22
73	69	59	43	23
74	70	60	44	23
75	71	61	44	23
76	72	61	45	23
77	73	62	45	24
78	74	63	46	24
79	75	64	46	24
80	76	65	47	25
81	77	66	48	25
82	78	66	48	25
83	79	67	49	26
84	80	68	49	26
85	81	69	50	26
86	82	70	51	27
87	83	70	51	27
88	84	71	52	27
89	85	72	52	28
90	86	73	53	28
91	87	74	54	28
92	87	74	54	28
93	88	75	55	29
94	89	76	55	29
95	90	77	56	29
96	91	78	56	30
97	92	78	57	30
98	93	79	58	30
99	94	80	58	31
100	95	81	59	31
—	12 & 10	13 & 9	14 & 8	15 & 7

21 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6	7	8	9	10	11
Nos.	·295	·563	·782	·931	·997	·975	·866	·680	·435	·149
1	0	1	1	1	1	1	1	1	0	0
2	1	1	2	2	2	2	2	1	1	0
3	1	2	2	3	3	3	3	2	1	0
4	1	2	3	4	4	4	3	3	2	1
5	1	3	4	5	5	5	4	3	2	1
6	2	3	5	6	6	6	5	4	3	1
7	2	4	5	7	7	7	6	5	3	1
8	2	5	6	7	8	8	7	5	3	1
9	3	5	7	8	9	9	8	6	4	1
10	3	6	8	9	10	10	9	7	4	1
11	3	6	9	10	11	11	10	7	5	2
12	4	7	9	11	12	12	10	8	5	2
13	4	7	10	12	13	13	11	9	6	2
14	4	8	11	13	14	14	12	10	6	2
15	4	8	12	14	15	15	13	10	7	2
16	5	9	13	15	16	16	14	11	7	2
17	5	10	13	16	17	17	15	12	7	3
18	5	10	14	17	18	18	16	12	8	3
19	6	11	15	18	19	19	16	13	8	3
20	6	11	16	19	20	20	17	14	9	3
21	6	12	16	20	21	20	18	14	9	3
22	6	12	17	20	22	21	19	15	10	3
23	7	13	18	21	23	22	20	16	10	3
24	7	14	19	22	24	23	21	16	10	4
25	7	14	20	23	25	24	22	17	11	4
26	8	15	20	24	26	25	23	18	11	4
27	8	15	21	25	27	26	23	18	12	4
28	8	16	22	26	28	27	24	19	12	4
29	9	16	23	27	29	28	25	20	13	4
30	9	17	23	28	30	29	26	20	13	4
31	9	17	24	29	31	30	27	21	13	5
32	9	18	25	30	32	31	28	22	14	5
33	10	19	26	31	33	32	29	22	14	5
34	10	19	27	32	34	33	29	23	15	5
35	10	20	27	33	35	34	30	24	15	5
36	11	20	28	34	36	35	31	24	16	5
37	11	21	29	34	37	36	32	25	16	6
38	11	21	30	35	38	37	33	26	16	6
39	12	22	30	36	39	38	34	27	17	6
40	12	23	31	37	40	39	35	27	17	6
41	12	23	32	38	41	40	36	28	18	6
42	12	24	33	39	42	41	36	29	18	6
43	13	24	34	40	43	42	37	29	19	6
44	13	25	34	41	44	43	38	30	19	7
45	13	25	35	42	45	44	39	31	20	7
46	14	26	36	43	46	45	40	31	20	7
47	14	26	37	44	47	46	41	32	20	7
48	14	27	38	45	48	47	42	33	21	7
49	14	28	38	46	49	48	42	33	21	7
50	15	28	39	47	50	49	43	34	22	7
—	**21**	**20**	**19**	**18**	**17**	**16**	**15**	**14**	**13**	**12**

21 TERMS — SINE θ

1 *is zero*

+	2	3	4	5	6	7	8	9	10	11
Nos.	·295	·563	·782	·931	·997	·975	·866	·680	·435	·149
51	15	29	40	47	51	50	44	35	22	8
52	15	29	41	48	52	51	45	35	23	8
53	16	30	41	49	53	52	46	36	23	8
54	16	30	42	50	54	53	47	37	23	8
55	16	31	43	51	55	54	48	37	24	8
56	17	32	44	52	56	55	48	38	24	8
57	17	32	45	53	57	56	49	39	25	8
58	17	33	45	54	58	57	50	39	25	9
59	17	33	46	55	59	58	51	40	26	9
60	18	34	47	56	60	59	52	41	26	9
61	18	34	48	57	61	59	53	41	27	9
62	18	35	48	58	62	60	54	42	27	9
63	19	35	49	59	63	61	55	43	27	9
64	19	36	50	60	64	62	55	44	28	10
65	19	37	51	61	65	63	56	44	28	10
66	19	37	52	61	66	64	57	45	29	10
67	20	38	52	62	67	65	58	46	29	10
68	20	38	53	63	68	66	59	46	30	10
69	20	39	54	64	69	67	60	47	30	10
70	21	39	55	65	70	68	61	48	30	10
71	21	40	56	66	71	69	61	48	31	11
72	21	41	56	67	72	70	62	49	31	11
73	22	41	57	68	73	71	63	50	32	11
74	22	42	58	69	74	72	64	50	32	11
75	22	42	59	70	75	73	65	51	33	11
76	22	43	59	71	76	74	66	52	33	11
77	23	43	60	72	77	75	67	52	33	11
78	23	44	61	73	78	76	68	53	34	12
79	23	44	62	74	79	77	68	54	34	12
80	24	45	63	74	80	78	69	54	35	12
81	24	46	63	75	81	79	70	55	35	12
82	24	46	64	76	82	80	71	56	36	12
83	24	47	65	77	83	81	72	56	36	12
84	25	47	66	78	84	82	73	57	37	13
85	25	48	66	79	85	83	74	58	37	13
86	25	48	67	80	86	84	74	58	37	13
87	26	49	68	81	87	85	75	59	38	13
88	26	50	69	82	88	86	76	60	38	13
89	26	50	70	83	89	87	77	61	39	13
90	27	51	70	84	90	88	78	61	39	13
91	27	51	71	85	91	89	79	62	40	14
92	27	52	72	86	92	90	80	63	40	14
93	27	52	73	87	93	91	81	63	40	14
94	28	53	74	88	94	92	81	64	41	14
95	28	53	74	88	95	93	82	65	41	14
96	28	54	75	89	96	94	83	65	42	14
97	29	55	76	90	97	95	84	66	42	14
98	29	55	77	91	98	96	85	67	43	15
99	29	56	77	92	99	97	86	67	43	15
100	30	56	78	93	100	98	87	68	44	15
−	**21**	**20**	**19**	**18**	**17**	**16**	**15**	**14**	**13**	**12**

21 TERMS — COS θ

1 *is* + 1·000

	+2	**+3**	**+4**	**+5**	**+6**	**−7**	**−8**	**−9**	**−10**	**−11**
Nos.	·956	·826	·623	·364	·075	·222	·500	·733	·901	·989
1	1	1	1	0	0	0	1	1	1	1
2	2	2	1	1	0	0	1	1	2	2
3	3	2	2	1	0	1	2	2	3	3
4	4	3	2	1	0	1	2	3	4	4
5	5	4	3	2	0	1	3	4	5	5
6	6	5	4	2	0	1	3	4	5	6
7	7	6	4	3	1	2	4	5	6	7
8	8	7	5	3	1	2	4	6	7	8
9	9	7	6	3	1	2	5	7	8	9
10	10	8	6	4	1	2	5	7	9	10
11	11	9	7	4	1	2	6	8	10	11
12	11	10	7	4	1	3	6	9	11	12
13	12	11	8	5	1	3	7	10	12	13
14	13	12	9	5	1	3	7	10	13	14
15	14	12	9	5	1	3	8	11	14	15
16	15	13	10	6	1	4	8	12	14	16
17	16	14	11	6	1	4	9	12	15	17
18	17	15	11	7	1	4	9	13	16	18
19	18	16	12	7	1	4	10	14	17	19
20	19	17	12	7	2	4	10	15	18	20
21	20	17	13	8	2	5	11	15	19	21
22	21	18	14	8	2	5	11	16	20	22
23	22	19	14	8	2	5	12	17	21	23
24	23	20	15	9	2	5	12	18	22	24
25	24	21	16	9	2	6	13	18	23	25
26	25	21	16	9	2	6	13	19	23	26
27	26	22	17	10	2	6	14	20	24	27
28	27	23	17	10	2	6	14	21	25	28
29	28	24	18	11	2	6	15	21	26	29
30	29	25	19	11	2	7	15	22	27	30
31	30	26	19	11	2	7	16	23	28	31
32	31	26	20	12	2	7	16	23	29	32
33	32	27	21	12	2	7	17	24	30	33
34	33	28	21	12	3	8	17	25	31	34
35	33	29	22	13	3	8	18	26	32	35
36	34	30	22	13	3	8	18	26	32	36
37	35	31	23	13	3	8	19	27	33	37
38	36	31	24	14	3	8	19	28	34	38
39	37	32	24	14	3	9	20	29	35	39
40	38	33	25	15	3	9	20	29	36	40
41	39	34	26	15	3	9	21	30	37	41
42	40	35	26	15	3	9	21	31	38	42
43	41	36	27	16	3	10	22	32	39	43
44	42	36	27	16	3	10	22	32	40	44
45	43	37	28	16	3	10	23	33	41	45
46	44	38	29	17	3	10	23	34	41	45
47	45	39	29	17	4	10	24	34	42	46
48	46	40	30	17	4	11	24	35	43	47
49	47	40	31	18	4	11	25	36	44	48
50	48	41	31	18	4	11	25	37	45	49
	+21	**+20**	**+19**	**+18**	**+17**	**−16**	**−15**	**−14**	**−13**	**−12**

1 *is* + 1·000

	+2	+3	+4	+5	+6	−7	−8	−9	−10	−11
Nos.	·956	·826	·623	·364	·075	·222	·500	·733	·901	·989
51	49	42	32	19	4	11	26	37	46	50
52	50	43	32	19	4	12	26	38	47	51
53	51	44	33	19	4	12	27	39	48	52
54	52	45	34	20	4	12	27	40	49	53
55	53	45	34	20	4	12	28	40	50	54
56	54	46	35	20	4	12	28	41	50	55
57	54	47	36	21	4	13	29	42	51	56
58	55	48	36	21	4	13	29	43	52	57
59	56	49	37	21	4	13	30	43	53	58
60	57	50	37	22	5	13	30	44	54	59
61	58	50	38	22	5	14	31	45	55	60
62	59	51	39	23	5	14	31	45	56	61
63	60	52	39	23	5	14	32	46	57	62
64	61	53	40	23	5	14	32	47	58	63
65	62	54	40	24	5	14	33	48	59	64
66	63	55	41	24	5	15	33	48	59	65
67	64	55	42	24	5	15	34	49	60	66
68	65	56	42	25	5	15	34	50	61	67
69	66	57	43	25	5	15	35	51	62	68
70	67	58	44	25	5	16	35	51	63	69
71	68	59	44	26	5	16	36	52	64	70
72	69	59	45	26	5	16	36	53	65	71
73	70	60	45	27	5	16	37	54	66	72
74	71	61	46	27	6	16	37	54	67	73
75	72	62	47	27	6	17	38	55	68	74
76	73	63	47	28	6	17	38	56	68	75
77	74	64	48	28	6	17	39	56	69	76
78	75	64	49	28	6	17	39	57	70	77
79	76	65	49	29	6	18	40	58	71	78
80	76	66	50	29	6	18	40	59	72	79
81	77	67	50	29	6	18	41	59	73	80
82	78	68	51	30	6	18	41	60	74	81
83	79	69	52	30	6	18	42	61	75	82
84	80	69	52	31	6	19	42	62	76	83
85	81	70	53	31	6	19	43	62	77	84
86	82	71	54	31	6	19	43	63	77	85
87	83	72	54	32	7	19	44	64	78	86
88	84	73	55	32	7	20	44	65	79	87
89	85	74	55	32	7	20	45	65	80	88
90	86	74	56	33	7	20	45	66	81	89
91	87	75	57	33	7	20	46	67	82	90
92	88	76	57	33	7	20	46	67	83	91
93	89	77	58	34	7	21	47	68	84	92
94	90	78	59	34	7	21	47	69	85	93
95	91	78	59	35	7	21	48	70	86	94
96	92	79	60	35	7	21	48	70	86	95
97	93	80	60	35	7	22	49	71	87	96
98	94	81	61	36	7	22	49	72	88	97
99	95	82	62	36	7	22	50	73	89	98
100	96	83	62	36	8	22	50	73	90	99
	+21	**+20**	**+19**	**+18**	**+17**	**−16**	**−15**	**−14**	**−13**	**−12**

TABLE FOR FINDING $\sqrt{a^2 + b^2}$

Let b be the smaller of the two : then $\sqrt{a^2 + b^2} = (a + \frac{1}{3}b) +$ following table.

a \ b	10	20	30	40	50	60	70	80	90	100
100	−2·8	−4·7	−5·6	−5·6	−4·9	−3·4	−1·2	+1·4	+ 4·5	+ 7·9
99	−2·8	−4·7	−5·6	−5·6	−4·8	−3·2	−1·0	+1·6	+ 4·7	+ 8·4
98	−2·8	−4·7	−5·5	−5·5	−4·7	−3·0	−0·8	+1·8	+ 5·0	+ 8·7
97	−2·8	−4·7	−5·5	−5·4	−4·6	−2·9	−0·6	+2·0	+ 5·3	+ 9·0
96	−2·8	−4·6	−5·4	−5·3	−4·5	−2·8	−0·4	+2·2	+ 5·6	+ 9·3
95	−2·8	−4·6	−5·4	−5·2	−4·4	−2·6	−0·3	+2·5	+ 5·9	+ 9·6
94	−2·8	−4·6	−5·3	−5·1	−4·3	−2·4	−0·1	+2·7	+ 6·1	+ 9·9
93	−2·8	−4·6	−5·3	−5·0	−4·1	−2·3	+0·1	+2·9	+ 6·4	+10·2
92	−2·8	−4·6	−5·2	−5·0	−4·0	−2·1	+0·3	+3·2	+ 6·7	+10·6
91	−2·8	−4·5	−5·2	−4·9	−3·9	−1·9	+0·5	+3·4	+ 7·0	+10·9
90	−2·7	−4·5	−5·1	−4·8	−3·7	−1·8	+0·7	+3·7	+ 7·3	+11·2
89	−2·7	−4·5	−5·1	−4·7	−3·6	−1·6	+0·9	+3·9	+ 7·6	
88	−2·7	−4·5	−5·0	−4·6	−3·5	−1·4	+1·1	+4·2	+ 7·9	
87	−2·7	−4·4	−5·0	−4·5	−3·3	−1·3	+1·3	+4·5	+ 8·2	
86	−2·7	−4·4	−4·9	−4·4	−3·2	−1·2	+1·5	+4·8	+ 8·5	
85	−2·7	−4·4	−4·9	−4·3	−3·1	−1·0	+1·8	+5·0	+ 8·8	
84	−2·7	−4·4	−4·8	−4·3	−2·9	−0·8	+2·0	+5·2	+ 9·1	
83	−2·7	−4·3	−4·8	−4·2	−2·8	−0·6	+2·3	+5·5	+ 9·4	
82	−2·7	−4·3	−4·7	−4·1	−2·7	−0·4	+2·5	+5·8	+ 9·7	
81	−2·7	−4·3	−4·7	−4·0	−2·6	−0·2	+2·7	+6·1	+10·0	
80	−2·7	−4·2	−4·6	−3·9	−2·4	0·0	+3·0	+6·4	+10·4	
79	−2·7	−4·2	−4·6	−3·8	−2·3	+0·2	+3·3	+6·7		
78	−2·7	−4·2	−4·5	−3·7	−2·1	+0·4	+3·5	+7·0		
77	−2·7	−4·1	−4·4	−3·5	−1·9	+0·6	+3·8	+7·3		
76	−2·7	−4·1	−4·3	−3·4	−1·8	+0·8	+4·0	+7·6		
75	−2·7	−4·1	−4·2	−3·3	−1·6	+1·0	+4·3	+7·9		
74	−2·7	−4·0	−4·2	−3·2	−1·5	+1·2	+4·6	+8·4		
73	−2·7	−4·0	−4·1	−3·0	−1·3	+1·5	+4·8	+8·7		
72	−2·7	−4·0	−4·0	−2·9	−1·1	+1·7	+5·1	+9·0		
71	−2·7	−3·9	−3·9	−2·8	−0·9	+1·9	+5·4	+9·3		
70	−2·6	−3·9	−3·8	−2·7	−0·7	+2·2	+5·7	+9·6		
69	−2·6	−3·9	−3·8	−2·6	−0·5	+2·4	+6·0			
68	−2·6	−3·8	−3·7	−2·4	−0·3	+2·7	+6·3			
67	−2·6	−3·8	−3·6	−2·3	−0·1	+2·9	+6·6			
66	−2·6	−3·7	−3·5	−2·2	+0·1	+3·2	+6·9			
65	−2·6	−3·7	−3·4	−2·0	+0·3	+3·5	+7·2			
64	−2·6	−3·6	−3·3	−1·9	+0·5	+3·8	+7·6			
63	−2·6	−3·6	−3·2	−1·7	+0·8	+4·0	+7·9			
62	−2·6	−3·5	−3·1	−1·5	+1·0	+4·3	+8·2			
61	−2·6	−3·5	−3·0	−1·4	+1·2	+4·6	+8·5			
60	−2·5	−3·4	−2·9	−1·2	+1·4	+4·9	+8·9			
59	−2·5	−3·4	−2·8	−1·1	+1·7	+5·2				
58	−2·5	−3·3	−2·7	−0·9	+1·9	+5·5				
57	−2·5	−3·3	−2·6	−0·7	+2·2	+5·8				
56	−2·5	−3·2	−2·4	−0·5	+2·4	+6·1				
55	−2·5	−3·2	−2·3	−0·3	+2·6	+6·4				
54	−2·5	−3·1	−2·2	−0·1	+2·8	+6·8				
53	−2·5	−3·1	−2·1	+0·1	+3·1	+7·1				
52	−2·5	−3·0	−2·0	+0·3	+3·4	+7·4				
51	−2·5	−2·9	−1·9	+0·5	+3·7	+7·7				
50	−2·3	−2·8	−1·7	+0·7	+4·0	+8·1				

Example (1). If $a = 77$ and $b = 73$

$$\begin{aligned} a + \tfrac{1}{3}b &= 101{\cdot}3 \\ \text{corr}^{n} &= +\ 4{\cdot}8 \\ \sqrt{a^2 + b^2} &= 106{\cdot}1 \end{aligned}$$

This is correct to 0·1, though the tables are not intended to be used beyond an accuracy of 1·0.

Example (2). If $a = 43$, $b = 29$

$$\begin{aligned} a + \tfrac{1}{3}b &= 52{\cdot}7 \\ \text{corr}^{n} &= -\ 0{\cdot}8 \\ &\ \underline{\underline{51{\cdot}9}} \end{aligned}$$

The correction is obtained by doubling a and b and then halving the correction given by the table.

To find q from $\sin q = b/\sqrt{a^2+b^2}$, where $b < a$.

Since q is always less than 45°, only the part of the table to the left of the thick broken line is for direct use: the rest of the table gives merely the proportional parts for interpolation.

$\sqrt{a^2+b^2}$ \ b	10	20	30	40	50	60	70	8	9	10
	°	°	°	°	°	°	°	°	°	°
100	5·7	11·5	17·5	23·6	30·0	36·9	44·5	4·8	5·4	6·0
99	5·8	11·6	17·6	23·8	30·3	37·3	45·0	4·8	5·5	6·1
98	5·9	11·8	17·8	24·0	30·7	37·7	4·3	4·9	5·5	6·1
97	5·9	11·9	18·0	24·3	31·0	38·2	4·3	5·0	5·6	6·2
96	6·0	12·0	18·2	24·6	31·4	38·7	4·4	5·0	5·7	6·3
95	6·0	12·2	18·4	24·9	31·8	39·2	4·5	5·1	5·8	6·4
94	6·1	12·3	18·6	25·2	32·2	39·7	4·5	5·1	5·8	6·4
93	6·2	12·4	18·8	25·5	32·6	40·2	4·6	5·2	5·9	6·5
92	6·2	12·5	19·0	25·8	33·0	40·7	4·6	5·3	5·9	6·6
91	6·3	12·7	19·2	26·1	33·4	41·2	4·7	5·4	6·0	6·7
90	6·4	12·8	19·5	26·4	33·8	41·8	4·7	5·4	6·0	6·7
89	6·5	13·0	19·7	26·7	34·2	42·4	4·8	5·5	6·1	6·8
88	6·5	13·1	19·9	27·0	34·6	43·0	4·8	5·5	6·2	6·9
87	6·6	13·2	20·2	27·4	35·1	43·6	4·9	5·6	6·3	7·0
86	6·7	13·4	20·5	27·7	35·5	44·2	5·0	5·7	6·4	7·1
85	6·8	13·6	20·7	28·0	36·0	44·9	5·0	5·8	6·5	7·2
84	6·8	13·8	21·0	28·4	36·5	4·4	5·1	5·8	6·6	7·3
83	6·9	14·0	21·3	28·8	37·0	4·4	5·2	5·9	6·7	7·4
82	7·0	14·1	21·6	29·2	37·5	4·5	5·3	6·0	6·8	7·5
81	7·1	14·3	21·9	29·6	38·1	4·6	5·3	6·1	6·9	7·6
80	7·2	14·5	22·2	30·0	38·7	4·6	5·4	6·2	7·0	7·7
79	7·3	14·7	22·5	30·5	39·3	4·7	5·5	6·3	7·1	7·9
78	7·4	14·9	22·8	30·9	39·9	4·8	5·6	6·4	7·2	8·0
77	7·5	15·0	23·0	31·3	40·5	4·9	5·7	6·5	7·3	8·1
76	7·6	15·3	23·3	31·8	41·1	4·9	5·7	6·6	7·4	8·2
75	7·7	15·5	23·6	32·2	41·8	5·0	5·8	6·7	7·6	8·4
74	7·8	15·7	23·9	32·7	42·5	5·1	5·9	6·8	7·7	8·5
73	7·9	15·9	24·2	33·2	43·2	5·1	6·0	6·9	7·8	8·6
72	8·0	16·1	24·6	33·7	44·0	5·2	6·1	7·0	7·9	8·8
71	8·1	16·3	25·0	34·3	4·4	5·3	6·2	7·1	8·0	8·9
70	8·2	16·6	25·4	34·8	4·5	5·4	6·3	7·2	8·1	9·0
69	8·3	16·8	25·8	35·4	4·6	5·5	6·4	7·3	8·2	9·1
68	8·5	17·1	26·2	36·0	4·6	5·5	6·4	7·4	8·3	9·2
67	8·6	17·3	26·6	36·6	4·6	5·6	6·5	7·4	8·4	9·3
66	8·7	17·6	27·0	37·3	4·7	5·6	6·6	7·5	8·5	9·4
65	8·8	17·9	27·5	38·0	4·8	5·7	6·7	7·6	8·5	9·5
64	9·0	18·2	27·9	38·7	4·8	5·8	6·7	7·7	8·6	9·6
63	9·1	18·5	28·4	39·5	4·8	5·8	6·8	7·8	8·7	9·7
62	9·3	18·8	28·9	40·2	4·9	5·9	6·9	7·8	8·8	9·8
61	9·4	19·1	29·4	41·0	5·0	6·0	6·9	7·9	8·9	9·9
60	9·6	19·5	30·0	41·8	5·0	6·0	7·0	8·0	9·0	10·0
59	9·8	19·9	30·6	42·6	5·1	6·1	7·1	8·2	9·2	10·2
58	9·9	20·2	31·2	43·5	5·2	6·2	7·3	8·3	9·4	10·4
57	10·1	20·6	31·8	44·5	5·3	6·4	7·4	8·5	9·5	10·6
56	10·3	21·0	32·4	4·3	5·4	6·5	7·6	8·6	9·7	10·8
55	10·5	21·4	33·0	4·4	5·5	6·6	7·7	8·8	9·9	11·0
54	10·7	22·0	33·7	4·5	5·6	6·7	7·8	9·0	10·1	11·2
53	10·9	22·4	34·4	4·6	5·7	6·8	8·0	9·1	10·3	11·4
52	11·1	22·8	35·2	4·6	5·8	7·0	8·1	9·3	10·4	11·6
51	11·3	23·2	36·0	4·7	5·9	7·1	8·3	9·4	10·6	11·8
50	11·5	23·6	36·9	4·8	6·0	7·2	8·4	9·6	10·8	12·0

Example: If $\sqrt{a^2+b^2} = 73$ and $b = 37$ look for 73 on left: in column 30 we find 24·2, and under 70 we find 6·0: total 30°·2, which is only 0°·3 in error. The table is not intended to give nearer than 1°.

EXAMPLE OF THE USE OF THE TABLES

The following example of the use of the tables is taken from a discussion (as yet unpublished) of the azimuth errors of the Cape Transit Circle for the years 1856–1904. Monthly mean errors were first formed, and since there was a conspicuous annual term, the mean value for each calendar month was subtracted from each individual value. Experience has shown that the extra labour of removing such conspicuous terms is well repaid. To keep the numbers chiefly positive a constant was added at the same time, or rather it was subtracted from the mean calendar values before applying them.

The residuals can now be grouped successively in any series of *n* terms: for example, 15 terms. Beginning with January, 1856, the first five groups of 15 terms are as follows (it is found convenient to take 5 periods together):

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	41	23	31	46	76	63	68	71	70	66	67	57	26	5	9
	36	45	48	60	59	42	36	42	32	6	−6	−2	25	45	43
	46	26	29	26	20	17	−6	−20	−11	19	65	53	37	23	12
	16	15	4	−17	−32	6	50	82	55	33	17	29	29	28	34
	17	14	63	79	99	61	43	22	15	19	14	21	15	29	66
Sums	56	23	75	94	122	89	91	97	61	43	57	58	32	30	64

The unit is $0^{s}\cdot01$. From the sums has been subtracted 100, so as to reduce them nearly all to two figures. (We could have subtracted 123, which would have removed the one exception: it is a matter of individual preference what constant to use, if any. As will appear below, it is needless to subtract any in dealing with an *even* number of terms.)

We have now to multiply these diminished sums by sin θ and cos θ, using the tables on pp. 16 and 17. But before doing so it is convenient to rewrite them as follows:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
	56	23	75	94	122	89	91	97	
	(56)	64	30	32	58	57	43	61	
Differences	0	−41	+45	+62	+64	+32	+48	+36	
Sines	0	−17	+33	+59	+64	+28	+28	+ 7	= +202
Sums	112	87	105	126	180	146	134	158	
Less 100	12	−13	5	26	80	46	34	58	
Cosines	6	−12	+3	+8	−8	−23	−28	−57	= −111

It will be seen from the headings of the columns on p. 16 that after column 8 the factors repeat in inverse order with negative sign: so that we can save arithmetic by taking differences. For the cosines we form the sums of the same pairs. If we again wish to omit a constant (such as 100) we must double the first term, which can be done by writing it in brackets before forming sums (or differences): but we must remember to use the factor ½ instead of unity in forming the products.

The formation of the products by use of the tables scarcely needs explanation.

The algebraical sums of the products must now be divided by D = 5 × 15/2, of which the 5 is due to the addition of 5 sets: and the 15/2 is $\Sigma \sin^2 \theta$, or $\Sigma \cos^2 \theta$. We then get

$$s = +202/\mathrm{D} = +5{\cdot}3 \qquad c = -111/\mathrm{D} = -3{\cdot}0$$

We may omit the decimal by changing the unit to $0^{s}{\cdot}001$.

Treating successive groups of 5 sets (of 15 terms each) in this way, we get the values of s and c shown in the first and second columns below. In the third column is given the value of $p = \sqrt{s^2 + c^2}$, which can either be found direct from a table of squares, or by use of the table on p. 36. In the fourth column is given $q = \tan^{-1} s/c$, where q can be found from the table on p. 37; or, if preferred, by dividing s/c and using a table of tangents.

s	c	p	q	Add	Sum	Q	$p \sin Q$	Integrations		$p \cos Q$
+ 53	−30	61	119°	0°	119°	+19°	+20	+20	−50	+ 58
− 85	− 6	86	266	144	50	−50	−66	−46	−70	+ 55
+ 18	−60	63	164	288	92	− 8	− 9	−55	−22	+ 62
+ 98	+33	103	71	432	143	+43	+70	+15	+33	+ 75
−136	−80	158	240	216	96	− 4	−11	+ 4	+18	+157
+ 22	−11	25	116	0	116	+16	+ 7	+11	+14	+ 24
− 24	+14	28	300	144	84	−16	− 8	+ 3	+ 3	+ 27
Sums					700		+97 −94			+458
Means					100°		±27			+ 65

The values of q may then be plotted on a diagram as on p. 41,* and we look for a straight line which shall pass through them all as nearly as possible. (Note the way in which the crosses are repeated at intervals of 360°.) In this particular case the slope of the straight line which fits best shows that consecutive values of q decrease by 144° approximately. Adding multiples of 144° (from which of course any multiple of 360° may be dropped), as in the fifth and sixth columns, we get a phase which is approximately constant at 100°, and, subtracting this mean value 100°, we get the residuals in column Q.

Forming $p \sin Q$, this is the component of the periodicity which should approximate to zero if we have identified the period and phase rightly. If our period is wrong, the values of $p \sin Q$ will slowly change: if our phase is wrong, they will not change, but the mean value will not be zero. We test the latter point by comparing the sums of the positive and negative terms. (They are nearly the same as it happens: but it might not have been so, for we determined our phase from consideration of q only, whereas we have now introduced p.)

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* The best form of subsequent procedure, which will be found on p. 42, only occurred to the editor of these tables after they were in type together with much of what follows. But since what had already been written throws some light on the procedure now recommended, it will first be reproduced.

To test whether the period is right, i.e. whether the value of $p \sin Q$ is slowly changing, a good method is to 'integrate' by adding successive terms: $+20-66$, $+20-66-9$, $+20-66-9+70$, &c. And having done this from the top, we may repeat it from below: $+3$, $+3+11$, $+3+11+4$, &c. These integrations are given in the ninth and tenth columns. Since the first number ($+20$) is positive and the last (-50) negative, the evidence is in favour of a slight decrease of $p \sin Q$; but it is so small and irregular that we may well rest satisfied with our first approximation. We have thus found a periodicity with coefficient $0^{s}{\cdot}065$ and period approximately

$$15\Big/\left(1+\frac{144}{5\times 360}\right)=15-1{\cdot}1=13{\cdot}9 \text{ months,}$$

since the phase changes* by 144° in five periods of 360°. We shall thus be nearer the true period when we divide our observations into groups of 14 months. But this periodicity is strong enough to dominate the arrangement in 15 months. It by no means follows that it is the only periodicity near 15 months, as we can best realize by removing it forthwith.

Adopted Phase	Calculated		Observed		Residuals		p_2	q_2	Add	Sum	Q_2	$p_2 \cos Q_2$
	s_1	c_1	s_1	c_1	s_2	c_2						
100°	+64	−11	+ 53	−30	−11	−19	22	210°	0°	210°	+66°	+ 9
316	−45	+45	− 85	− 6	−40	−51	64	219	−110	109	−35	+52
172	+ 9	−64	+ 18	−60	+ 9	+ 4	10	66	−220	206	+62	+ 5
28	+31	+57	+ 98	+33	+67	−24	72	110	−330	140	− 4	+72
244	−59	−29	−136	−80	−77	−51	92	236	− 80	156	+12	+90
100	+64	−11	+ 22	−11	−42	0	42	270	−190	80	−64	+18
316	−45	+45	− 24	+14	+21	−31	37	146	−300	206	+62	+18

The adopted phase in the first column is simply $q-Q$: and using the coefficient 65 we form the theoretical values of s and c in the 2nd and 3rd columns. Subtracting them from the observed values we get s_2 and c_2 for discussion of other periodicities. Proceeding exactly in the same way as before we find that an advance of phase of about 110° will suit the observations; giving a period of

$$15\Big/\left(1-\frac{110}{5\times 360}\right)=15{\cdot}98 \text{ months.}$$

We might now remove this term and proceed to look for others; and we shall find others present themselves by their accordance of phase, i.e. by the possibility of drawing straight lines through the points on a diagram similar to those on p. 41. The reason for the successive emergence of new periods will be seen more clearly when we come to consider the Fourier sequence of exact submultiples. Each member of this series is wholly independent of the others, and remains intact when the others are removed. Hence, if we remove the big terms, the smaller terms

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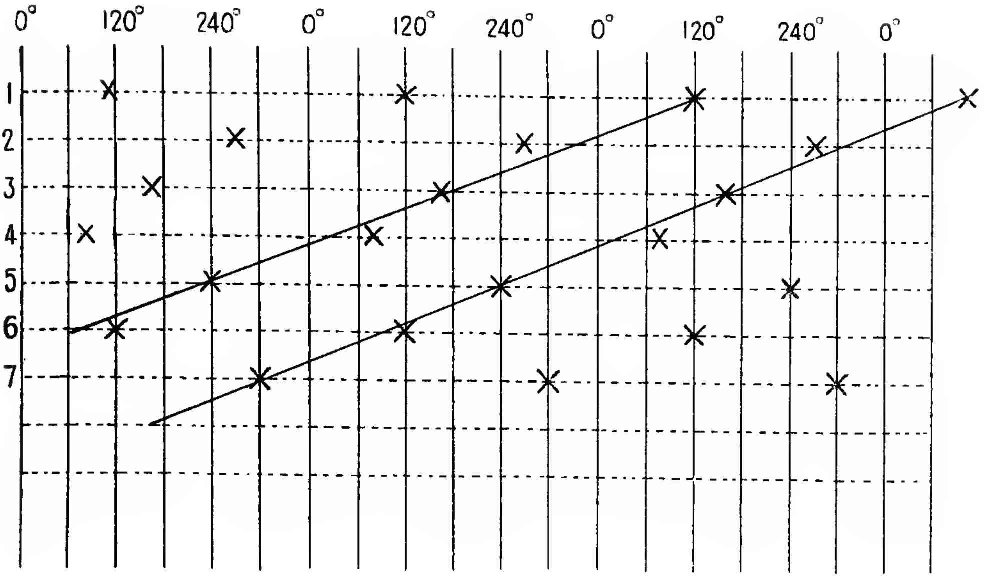
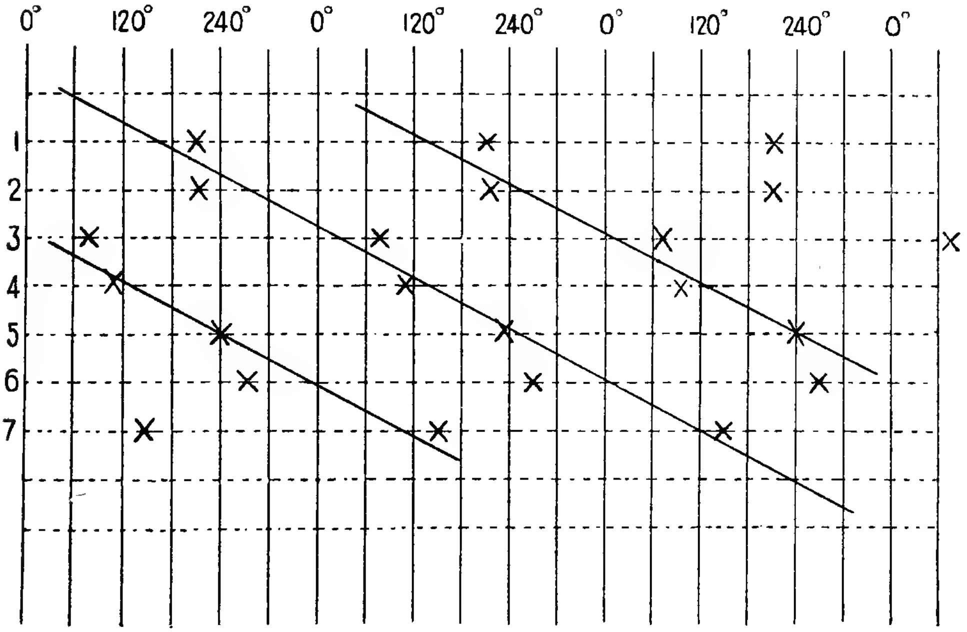
* The change is not strictly uniform, but we may neglect the variations in this exploring work.

EXAMPLE OF THE USE OF THE TABLES





remain and manifest themselves by an accordance of phase which may be even striking, but is not significant of reality. It merely means that the larger terms have been removed. We proceed, therefore, to the exposition of this sounder method, which had not been reached until the foregoing was in type.

DEFINITIVE PROCEDURE

Suppose the whole available period of observation divided into exact submultiples. In the present case we have observations (of azimuth of the Cape Transit Circle) for the years 1856–1904, i. e. 49 years or 588 months. We need not write down the whole series of submultiples: those which concern us in the example under consideration are near 15 months, and it will suffice to consider the

Divisors	36	37	38	39	40	41	42	43
Submultiples	16·33	15·89	15·48	15·08	14·70	14·34	14·00	13·68 months

The coefficients of sin θ and cos θ should be determined for these periods, and for these only: and the information published in this form.

It is not absolutely necessary to calculate all the above in connexion with the series in 15 months: the first two are nearer 16 months, and the last three nearer 14. But it will be a useful check on the work to include these overlaps.

The reasons for selecting these definite periods cannot be fully given here; but briefly they may be thus summarized:

(*a*) The investigation of all these periods is *necessary*. Fourier showed that the coefficients are all independent of one another: hence no one can be inferred from the others.

(*b*) It is also *sufficient*: it can be shown that the coefficients for any intermediate period can be deduced from these standard constituents.

To these essential reasons may be added:

(*c*) That the procedure is *instructive*: for instance, we see at once why the elimination of one such periodicity (as 13·9 months in the above example) is liable to reveal the existence of others.

(*d*) Finally, the procedure will be found convenient. For instance, it avoids the uncertainty of that above sketched.

We will now apply it to the above example. Suppose we are to deal with a true periodicity of 16 months exactly. Since we have cut up the series into lengths of 15 months, successive maxima will fall one month later, until after *sixteen* periods of 15 months each the maximum returns to the same place: the phase thus changes by $360°/16$ each period, or by $5 \times 360°/16$ in five periods. Generally, if the period to be dealt with is $15 + x$ months, the change of phase is

$$5x \times 360°/(15 + x)$$

which for the periods above enumerated has the values

Divisor	36	37	38	39	40	41	42	43
Submultiple	16·33	15·89	15·48	15·08	14·70	14·34	14·00	13·68
Advance	+146°	+100°	+56°	+10°	−37°	−83°	−129°	−174°

The calculation of the coefficients may now be arranged as follows :

q	Add	Sum $= Q_1$	p	$S_1 = p \sin Q_1$	$C_1 = p \cos Q_1$	Add	Sum $= Q_2$	$S_2 = p \sin Q_2$	$C_2 = p \cos Q_2$
119°	− 73°	46°	61	+ 44	+42	− 50°	69°	+ 57	+ 22
266	−219	47	86	+ 63	+59	−150	116	+ 77	− 38
164	− 5	159	63	+ 23	−59	−250	274	− 63	+ 4
71	−151	280	103	−101	+18	−350	81	+102	+ 16
240	−297	303	158	−133	+86	− 90	150	+ 79	−137
116	− 83	33	25	+ 14	+21	−190	286	− 24	+ 7
300	−229	71	28	+ 26	+ 9	−290	10	+ 5	+ 28
			Mean	− 9	+25			+ 33	− 14

In the first column q is repeated from p. 39 : in the second successive multiples of 146° (which is the advance for divisor 36) are written down, beginning, however, with 146°/2 to reduce from the mean of the first group to the beginning of it. [It will clearly be convenient to reduce all our terms to the same epoch, and the time of the first observation is hereby adopted as that epoch.] In the third column Q_1 is formed as the sum of the first two: then p is repeated from p. 39: and $p \sin Q_1$, $p \cos Q_1$ are formed in the next two columns. The remainder of the columns give the formation of $p \sin Q_2$ and $p \cos Q_2$ in the same way, using now 100°, the advance for divisor 37. The whole set of values works out as follows :

Divisor	36	37	38	39	40	41	42	43
Period	16·33	15·89	15·48	15·08	14·70	14·34	14·00	13·68
S	− 9	+33	− 8	+ 5	−10	− 9	+53	−48
C	+25	−14	+ 8	−25	+11	+15	−32	+ 7
$\sqrt{S^2+C^2}$	27	36	11	26	15	18	62	49

Before considering the significance of these figures we may notice two sources of inaccuracy. Firstly, these terms should each run through an exact number of periods, and if we had used the whole series of observations this would be the case. But if we stop at the end of the last complete set of 15 months, and all the more if we stop at the end of the last complete set of $5 \times 15 = 75$ months, some observations are not used. In the present example we only use $7 \times 75 = 525$ months out of the whole 588 : hence the coefficients deduced by this summary process will be to that extent inaccurate. But since the process is essentially one of exploration, to be supplemented by more exact methods (e.g. the use of 3 or more figures instead of 2) where necessary, this defect is not serious ; and there are several tolerably obvious remedies, if it is desired to apply one.

Secondly, grouping five periods together depresses the coefficient of a periodicity which differs from 15 months exactly. Suppose that the change of phase is ϕ per period, or $5\ \phi$ in 5 periods. Then a set of 5 periods will give approximately the terms :

$$q \cos(\theta_0 - 2\ \phi) + q \cos(\theta_0 - \phi) + q \cos \theta_0 + q \cos(\theta_0 + \phi) + q \cos(\theta_0 + 2\ \phi)$$
$$= q \cos \theta_0 \left[1 + 2 \cos \phi + 2 \cos 2\ \phi\right]$$

The value of the factor $(1 + 2 \cos \phi + 2 \cos 2\ \phi)/5$ is given in the following table :

$5\ \phi$.	Factor.	$5\ \phi$.	Factor.	$5\ \phi$.	Factor.
20°	1·00	100	·88	180	·65
40°	·98	120	·83	200	·58
60°	·96	140	·78	220	·50
80°	·92	160	·72	240	·43

The coefficients above determined should therefore be divided by the appropriate factors from the above table, and accordingly take the values :

Divisor	36	37	38	39	40	41	42	43
S	−12	+38	− 8	+ 5	−10	−10	+64	−72
C	+33	−16	+ 8	−25	+11	+17	−40	+10
$\sqrt{S^2+C^2}$	36	41	11	26	15	20	76	74

It will be seen that both these sources of inaccuracy are practically removed if we work with single periods instead of groups of 5. It is easy to apply this more rigorous procedure when we have made our first rough survey to see where the important periodicities lie.

Returning now to the figures themselves, it is clear that the important terms are those for which $\sqrt{S^2+C^2}$ is large. But are the true periodicities these terms themselves or some combination of them ? This question raises some fundamental issues, especially :

(a) *What is meant by a real periodicity when it is clearly not single and isolated ?*

Apart from the planetary motions periodicities in nature are seldom clear cut. Thus, there is undoubtedly an annual periodicity in temperature; but its coefficient and phase both vary within limits, so that when we analyse a long series of observations we get coefficients for other periods near one year, which may conceivably be quite large. It is also conceivable that an undoubtedly real term (due to a *vera causa* with period differing slightly from a tropical year—say a sidereal year) should be mixed up with these. No criterion has yet been established, or even suggested, for distinguishing these alternatives *from the periodogram* (*or its equivalent*) *alone.* Of course, if we have independent knowledge of the *vera causa*, the situation is wholly altered.

(b) *How closely can we identify the period of a periodicity not single and isolated?*

It is believed by the editor, as the result of much work in this field, that the closeness has hitherto been much over-estimated, and the uncertainty may extend over wide limits. This will appear from what follows.

For reasons such as these it is suggested that the straightforward work ends with the formation of the coefficients for the Fourier sequence as above. When the material is once presented in this form, it is readily available for discussion (as will now be shown): but for full discussion other knowledge may be, or even must be, made use of; the problem ceases to be one of dealing with the given observations alone, and we have accordingly reached a definite stage. It is not the end, but it is a necessary stopping-place.

For further discussion it is, however, necessary to consider what happens when there is a real periodicity falling between two of those tabulated. Suppose it falls between those corresponding to divisors r and $r + 1$: denote the corresponding divisor by $r + x$, where $x < 1$. Then it will affect *all* the terms tabulated, but more especially r and $r + 1$. Both the S and C series are affected in the same manner, and the following are the approximate factors multiplying the coefficient (whether of S or C) to get the coefficients of the Fourier terms.

Divisor	$r-2$	$r-1$	r	$r+1$	$r+2$	$r+3$
Factor	$+\frac{\text{Sin}\,\pi x}{\pi(x+2)}$	$+\frac{\text{Sin}\,\pi x}{\pi(x+1)}$	$+\frac{\text{Sin}\,\pi x}{\pi x}$	$-\frac{\text{Sin}\,\pi x}{\pi(1-x)}$	$-\frac{\text{Sin}\,\pi x}{\pi(2-x)}$	$-\frac{\text{Sin}\,\pi x}{\pi(3-x)}$
$x = 0$	·00	·00	+1·00	·00	·00	·00
$x = 1/6$	+·07	+·14	+ ·95	−·19	−·09	−·06
$x = 1/3$	+·12	+·21	+ ·82	−·41	−·16	−·10
$x = 1/2$	+·13	+·21	+ ·63	−·63	−·21	−·13
$x = 2/3$	+·10	+·16	+ ·41	−·82	−·21	−·12
$x = 5/6$	+·06	+·09	+ ·19	−·95	−·14	−·07
$x = 1$	·00	·00	·00	−1·00	·00	·00

The mode of formation of the factors in the second row is tolerably obvious: and the numerical values given for a few selected values of x show the practical extent of the influence, which is only confined to a single term when the period has been hit off with great precision.

The striking feature in general is the *change of sign*. When a real periodicity lies between two Fourier terms, these terms must have coefficients of opposite signs, both in the S and C series. Looking at our example we see that this happens to be so in every case but that of 40–41: so that we might look for half a dozen periodicities. But clearly *some* of these are accidental. If we proceed to inquire which are accidental and which real we find great difficulties in drawing the line. Several alternative criteria might be suggested, but the discussion of their merits would lead us into the discussion which has been already relegated elsewhere. We may, however, note the difficulty, already referred to, of identifying a periodicity closely. There is clearly a noteworthy periodicity near divisor 42, or 14·00 months. But on which side is it? There are notable changes of sign on both sides of 42 : if we stick to the notion of a single periodicity we must neglect something as accidental error, and the choice of what we may neglect is wide. If we admit two periodicities in this neighbourhood, then by sharing the middle coefficient differently between them, we can vary the selected periods considerably. Other independent knowledge might immediately limit our choice ; that is all that can be said.

If, however, we have such independent knowledge of the period, and it falls at $(r + x)$, then we can build up its coefficients from those tabulated by use of the table already given. Suppose the period is known to be 14·11 months, corresponding to divisor $41\frac{2}{3}$: so that $r = 42$ and $x = 2/3$. It will suffice to consider S only, and we take its corrected values :

Divisor	36	37	38	39	40	41	42	43
S	−12	+38	− 8	+ 5	−10	−10	+64	−72

The contribution from 41 is − 10 × − ·41 = + ·041
The contribution from 40 is − 10 × − ·16 = + ·016
The contribution from 39 is + 5 × − ·10 − ·005

and so on : total about + ·05 on the negative side. On the positive side we have not as yet sufficient information, since the series does not extend beyond 43. But so far as it goes we have

$$+ \cdot 64 \times \cdot 82 - \cdot 72 \times \cdot 21 = + \cdot 37$$

and the calculation can be completed when the periods near 14, 13, 12 . . . months have been duly examined.

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